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To learn control 1/f noise in electronic devices, the author's Quantum 1/f Noise theory was further developed and applied to pn junctions, junction and MIS infrared detectors, bipolar transistors, FET, BJT, vacuum tubes, secondary emission tubes, SQUIDs, and other devices. The present report gives a review of the progress made in the theory of the Quantum 1/f Effect, including the general derivation of the effect in second quantization, the derivation of quantum 1/f cross-correlations, the effect of a finite mean free path in condensed matter, the characteristic functional, piezoelectric coherent quantum 1/f noise, and an interpolation formula for coherent and conventional quantum 1/f noise. The practical applications presented here are limited here to infrared detectors and SQUIDs, the rest are referenced. Optimal low-noise design principles based on the 1/f theory are formulated.

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FUNDAMENTAL QUANTUM 1/F NOISE IN ULTRASMALL SEMICONDUCTOR DEVICES AND THEIR OPTIMAL DESIGN PRINCIPLES

FINAL TECHNICAL REPORT

Grant: AFOSR-85-0130; Period: May 1, 1985 - April 31, 1989

by

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ABSTRACT

The Quantum 1/f Theory has been developed under the present Grant and has been applied to the noise suppression in various electronic devices. The Quantum 1/f Theory derives fundamental quantum fluctuations present in the elementary processes of physics at the level of the quantum mechanical cross sections and process rates. New developments described in this report include the derivation of the conventional Quantum 1/f Effect in second quantization, and for an arbitrary number of particles in the final state. They also include the derivation of the quantum 1/f cross-correlations between scattering cross section fluctuations at different angles. These cross-correlations are needed for a more exact calculation of quantum 1/f noise in various kinetic coefficients, such as the mobility of the current carriers. Another new development is the derivation of a formula which describes the effect of the finite mean free path of the current carriers in condensed matter on the quantum 1/f noise. Finally, an interpolation formula linking conventional and coherent quantum 1/f noise was developed. Practical applications to semiconductor samples, pn junctions, junction and MIS detectors, Josephson junctions and SQUIDs were developed and are presented. Optimal design principles based on the Quantum 1/f Theory have been developed and are described, applied and explained.

FUNDAMENTAL QUANTUM 1/F NOISE IN ULTRASMALL SEMICONDUCTOR DEVICES AND THEIR OPTIMAL DESIGN PRINCIPLES

CONTENTS

I. INTRODUCTION

Under the present Grant major progress was achieved by the author both in developing the Quantum 1/f Theory[1-13], and in applying it to high-technology systems[16-20], in particular to low-noise design of electronic devices, including ultrasmall devices. The present report reviews this progress, presents the physical meaning of the Quantum 1/f Effect, and discusses the impact it has both on practical hi-tech applications and on the foundations of quantum physics.

The Quantum 1/f Effect was first derived formally by the author in second quantization for a pair of bosons or fermions in 1986. In 1987 this second quantization treatment was extended by the author to an arbitrary number N of bosons or fermions. This allowed for the first time a rigorous derivation of the 1/N factor present in the Quantum 1/f Theory. Before 1986, the simplified physical derivations[1,2] of 1975 and 1980 were used, similar to the elementary theory of diffraction, translated into the time domain. The present *second-quantized formulation* of the theory is physically equivalent to the previous formulation, but is more rigorous, while still easily accessible. The present formulation *justifies the use of* four single-particle wave functions in the autocorrelation functions, provides a theoretical basis for the simplified elementary model based on interference beats between the bremsstrahlung loss parts and the non-bremsstrahlung part of the wave function, and clarifies the basic role of *quantum exchange* between identical particles in the Quantum 1/f Effect. This present formulation is briefly presented in Sec. II.2 and II.3, and is described in more detail in a review of the starting points of the Quantum 1/f Theory submitted to Physical Review. Strictly speaking, the derivation presented in Sec. II.2 and II.3 calculates the quantum 1/f fluctuations in space along the scattered beam in a scattering experiment, and these fluctuations are translated into fluctuations in time by considering the passage of the waves past a fixed point in space. A *direct derivation in time and space* is presented in Sec. II.5 for N=2, and yields the same result, thereby justifying the translation of wave number spectra into frequency spectra, applied in Sec. II.2-3.

Important progress was achieved through an extension of the second -quantized derivation to include the *cross-correlations,* as shown in Sec. II.6. These are correlations between the quantum 1/f fluctuations which a physical cross section or process rate presents for scattering in a certain direction, and the similar fluctuations observed on the same cross section or process rate in another direction. Knowledge of these cross-correlations is necessary for the calculation of the resultant mobility fluctuations in condensed matter. A preliminary calculation of the resulting mobility fluctuations was published by Kousik et al.[9] in 1986, yielding good agreement with the experiment, and is now being revised and refined by us on the basis of these more rigorous cross-correlation results.

Another important problem in which significant progress was made, concerns the *effect of a finite mean free path* on quantum 1/f noise in condensed matter. In principle, the Quantum 1/f Effect affecting a cross section in condensed matter should differ from the effect calculated for the same cross section in isolated conditions. The calculation presented in Sec. II.7 shows, indeed, some differences expressed in terms of a correction factor $(1-m^2)^{-1}$ which takes into account multiple scattering with a memory coefficient $0<m<1$ in each individual scattering process. However, the correction factor turns out to be 1 in the case of ideally randomizing collisions ($m=0$) which erase completely the memory of the incoming particle momentum (e.g., lattice scattering), and which provide most of the quantum 1/f noise anyway. This proves that all our earlier calculations, which were equivalent with neglecting this correction, did not overestimate the quantum 1/f noise, and were an excellent approximation, in spite of the skepticism of some critics.

The characteristic functional is known to provide the most complete description of a random process. The characteristic functional of quantum 1/f noise was calculated in 1982. However, the Quantum 1/f Effect also affects the thermal noise, introducing a non-Gaussian component. Under the present Grant the author was able to calculate the characteristic functional of physical thermal noise for the first time, as we briefly report in Sec. II.8.

An important problem related to 1/f nose in small and ultrasmall devices is the transition between conventional quantum 1/f noise which is applicable to ultrasmall devices, and coherent state quantum 1/f noise which is applicable in the limit of large devices. Due to the complexities of calculating infrared radiative corrections for correlations in many-body systems in condensed matter conditions, the progress achieved on this problem during the grant period was unsatisfactory and limited to the creation of a heuristic interpolation formula based on the author's physical picture which associates the coherent quantum 1/f noise with the magnetic field of the collective drift motion of the carriers. While the interpolation formula makes sense physically, it also is in general agreement with the experiment, but it does not represent a mathematical solution. This interpolation formula is presented at the end of Sec. II.9.

The whole Quantum 1/f Theory can be reformulated by replacing photons with piezophonons as the infraquanta. This also includes coherent state quantum 1/f noise. Under the present grant the coherent piezoelectric quantum 1/f noise was also studied and is briefly presented in Sec. II.10. More experiments are needed to verify it in detail, although it is known that it provides in principle an explanation of the giant Hooge parameters observed in ferroelectric materials.

Along with the further development of the quantum 1/f theory, a major emphasis was placed on the application to semiconductor devices, which is presented in part in Sec. III. The method for calculating mobility fluctuation Hooge parameters for various scattering processes in semiconductors was first outlined in the author's 1982 ONR Final Technical Report[37]. This method was applied in detail by Kousik et al.[9] and resulted in good agreement with the experimental data for semiconductor samples, and even with the corresponding Hooge parameters estimated without averaging over the energy bands. A general introduction to the application to pn junctions and MIS devices is provided in Sec. III.2.

The Quantum 1/f Theory was first applied to pn junctions by van der Ziel and Handel[16] in 1985, and was applied to n^+p junction infrared detectors[17] by van der Ziel, Handel, Wu and Anderson in 1986 and[18] 1989. This later work is presented in Sec. III.3, and led to the formulation of

optimal design principles which have contributed to the practical improvement of infrared detectors. In 1987 the Quantum 1/f Theory was also applied by the author to MIS infrared detectors[20], as shown in Sec. III. 4, leading to analytical formulae which allow the calculation of quantum 1/f noise in all components of the device current and in the resulting detector performance. This should allow for practical MIS device optimization.

In cooperation with A. van der Ziel the Quantum 1/f Theory was used in 1986 to calculate the Hooge coefficient of $Hg_{1-x}Cd_xTe$, including a relativistic correction in the Umklapp scattering part[19]. The correction is needed, because the very low effective mass of electrons in narrow band gap $Hg_{1-x}Cd_xTe$ leads to speeds close to the speed of light, for the crystal momenta usually encountered in Umklapp scattering processes. Also in cooperation with van der Ziel, the quantum partition 1/f noise in vacuum pentodes[21] was reconsidered and shown again to be in agreement with the experimental data. The theory was also successfully applied by van der Ziel to, and verified in, secondary emission[22] and photomultiplier tubes, in vacuum photodiodes[13], silicon pn diodes[23], FETs[13], JfETs[13] BJTs[24]

SQUIDs are important high-tech devices limited in their performance by 1/f noise. Therefore, an application of the Quantum 1/f Theory to Josephson junctions and SQUIDs was performed[25], based on the calculation of quantum 1/f fluctuations in the normal resistance value of the junction. These fluctuations are reduced in turn to the Quantum 1/f Effect present in the elementary cross sections which define the normal resistance, and are calculated with the universal quantum 1/f formula. The normal resistance enters into the formula of the critical current of the Josephson junctions. Very good agreement with the experiment is obtained. This application is briefly presented in Sec. III.5.

The few selected applications presented in more detail in this report will illustrate the practical use of the quantum 1/f formulae, and the fruitful interaction between theory and experiment in the genesis of this new discipline linking electrical engineering and electrophysics with quantum electrodynamics.

II. QUANTUM 1/f NOISE THEORY

II.1. SIMPLE PHYSICAL DERIVATION OF QUANTUM 1/f NOISE

The Quantum 1/f Effect, also known as conventional quantum 1/f noise[1-10], is a fundamental fluctuation of physical cross sections and process rates, caused by the infrared-divergent coupling of current carriers to low frequency photons and other infraquanta, e.g., transversal phonons with piezoelectric coupling, or electron - hole pairs on the Fermi surface of metals. The physical origin of quantum 1/f noise is easy to understand. Consider for example Coulomb scattering of current carriers, e.g., electrons on a center of force. The scattered electrons reaching a detector at a given angle away from the direction of the incident beam are described by DeBroglie waves of a frequency corresponding to their energy. However, some of the electrons have lost energy in the scattering process, due to the emission of Bremsstrahlung. Therefore, part of the outgoing DeBroglie waves is shifted to slightly lower frequencies. When we calculate the probability density in the scattered beam, we obtain also cross terms, linear both in the part scattered with and without bremsstrahlung. These cross terms oscillate with the same frequency as the frequency of the emitted bremsstrahlung photons. The emission of photons at all frequencies results therefore in probability density fluctuations at all frequencies. The corresponding current density fluctuations are obtained by multiplying the probability density fluctuations by the velocity of the scattered current carriers. Finally, these current fluctuations present in the scattered beam will be noticed at the detector as low frequency current fluctuations, and will be interpreted as fundamental cross section fluctuations in the scattering cross section of the scatterer. Although the wave function φ of each carrier is split into a Bremsstrahlung part and a non-Bremsstrahlung part, no quantum 1/f noise can be observed from a single carrier. A single carrier will only provide a pulse in the detector. Many carriers are needed to produce the quantum 1/f noise effect, just as in the case of electron diffraction patterns, where each individual particle is diffracted, but

unless we repeat the experiment many times, or use many particles, no diffraction pattern can be seen. A single particle only yields a point of impact on the photographic plate in diffraction, or a pulse in the detector in the 1/f noise case. While incoming carriers may have been Poisson distributed, the scattered beam will exhibit super-Poissonian statistics, or bunching, due to this new effect which we may call quantum 1/f effect. The quantum 1/f effect is thus a many-body or collective effect, at least a two-particle effect, best described through the two-particle wave function and two-particle correlation function.

Let us estimate the magnitude of the quantum 1/f effect by starting with the classical (Larmor) formula $2q^2\mathbf{a}^2/3c^3$ for the power radiated by a particle of charge q and acceleration $\mathbf{a}$. The acceleration can be approximated by a delta function $\mathbf{a}(t) = \Delta\mathbf{v}\delta(t)$ whose Fourier transform $\Delta\mathbf{v}$ is constant and is the change in the velocity vector of the particle during the almost instantaneous scattering process. The one-sided spectral density of the emitted Bremsstrahlung power $2q^2(\Delta\mathbf{v})^2/3c^3$ is therefore also constant. The number $2q^2(\Delta\mathbf{v})^2/3hfc^3$ of emitted photons per unit frequency interval is obtained by dividing with the energy hf of one photon. The probability amplitude of photon emission $[2q(\Delta\mathbf{v})^2/3hfc^3]^{1/2}e^{i\gamma}$ is given by the square root of this photon number spectrum, including also a phase factor $e^{i\gamma}$. Let ψ be the Schrödinger wave function of the scattered outgoing charged particles, which can be constructed from single-particle wave functions. The beat term in the probability density $|\psi|^2$ is linear both in this Bremsstrahlung amplitude and in the non-Bremsstrahlung amplitude. Its spectral density will therefore be given by the product of the squared probability amplitude of photon emission (proportional to 1/f) with the squared non-Bremsstrahlung amplitude which is independent of f. The resulting spectral density of fractional probability density fluctuations is obtained by dividing with $|\psi|^4$ and is therefore

$$|\psi|^{-4}S_{|\psi|^2}(f) = 8q^2(\Delta\mathbf{v})^2/3hfNc^3 = 2A/fN = j^{-2}S_j(f), \qquad (1)$$

where $\alpha = e^2/\hbar c = 1/137$ is the fine structure constant and $\alpha A = 4q^2(\Delta\mathbf{v})^2/3\hbar c^3$ is known as the infrared exponent in quantum field theory,

and is known as the quantum 1/f noise coefficient, or Hooge constant, in electrophysics.

The spectral density of current density fluctuations is obtained by multiplying the probability density fluctuation spectrum with the squared velocity of the outgoing particles. When we calculate the spectral density of fractional fluctuations in the scattered current j, the outgoing velocity simplifies, and therefore Eq. (1) also gives the spectrum of current fluctuations $S_j(f)$, as indicated above. The quantum 1/f noise contribution of each carrier is independent, and therefore the quantum 1/f noise from N carriers is N times larger; however, the current j will also be N times larger, and therefore in Eq. (1) a factor N was included in the denominator for the case in which the cross section fluctuation is observed on N carriers simultaneously.

The fundamental fluctuations of cross sections and process rates are reflected in various kinetic coefficients in condensed matter, such as the mobility μ and the diffusion constant D, the surface and bulk recombination speeds s, and recombination times τ, the rate of tunneling j_t and the thermal diffusivity in semiconductors. Therefore, the spectral density of fractional fluctuations in all these coefficients is given also by Eq. (1). This is true in spite of the fact that each carrier will undergo many consecutive scattering processes in the diffusion process. The quantum 1/f noise in the mobility and in the diffusion coefficient is practically the same[8] as the quantum 1/f noise in a single representative scattering event which limits the mobility or the diffusion coefficient.

Due to the rapid relaxation of concentration fluctuations, the quantum 1/f fluctuations of scattering cross sections will only be reflected by the fluctuations of the mobility and the diffusion constant of the carriers, and not by fluctuations in the concentration of carriers.

For large devices the concept of coherent state quantum 1/f noise was introduced[11, 12]. In this case the Hooge parameter α_H may be written

$$\alpha_H = (\alpha_H)_{coh} = 2\alpha/\pi = 4.6\ 10^{-3}, \qquad (2)$$

where $\alpha = 1/(137)$ is the fine structure constant. This is of the same order of magnitude as the empirical value $\alpha_H = 2\ 10^{-3}$ that Hooge found for

long devices. It is therefore proposed that Hooge's empirical value for α_H is due to coherent state quantum 1/f noise, so that it has a very fundamental origin.

For small devices (e.g., of size L < 10 μ) we apply conventional, or incoherent[1-10], quantum 1/f noise which is just the cross section fluctuation introduced above in Eq. (1). In that case α_H may be written

$$\alpha_H = (\alpha_H)_{incoh} = (4\alpha/3\pi)[(\Delta v)^2/(c^2)], \qquad (3)$$

where Δv is the change in the velocity of the carriers in the interaction process considered. This expression holds for any 1/f noise source describable by fluctuating cross sections. Since usually $(\Delta v/c)^2 << 1$, except for carriers with a very small effective mass, we now have $\alpha_H <$ 3.1 10^{-3}. This may explain the low values of α_H (in the range of $\alpha_H = 10^{-5} - 10^{-9}$) for very small devices. In between one can introduce a parameter $s = f(L/L_o)$ where L_o is a characteristic size and write[12].

$$\alpha_H = (\alpha_H)_{incoh}[1/(1 + s)] + (\alpha_H)_{coh}[s/(1 + s)], \qquad (4)$$

with $s << 1$ for $L/L_o << 1$ and $s >> 1$ for $L/L_o >> 1$. This describes the transition from Eq. (2) to Eq. (3) when one goes to devices with smaller and smaller sizes. A suggestion for the calculation of s was presented by the author at the Rome 1985 Conf. on Noise in Physical Systems and 1/f Noise, $s = 2e^2N'/m^*c^2 = N' \bullet 5.5 \bullet 10^{-13}$cm, where N' is the number of carriers per unit length of the sample or device in the direction of current flow, and $5.5 \bullet 10^{-13}$cm is twice the classical radius of the electron. According to this rough approximation[12], $L_o = 100\mu$ for samples with a concentration c of carriers of $10^{15}cm^{-3}$ and varies proportional to $c^{-1/2}$.

When we apply Eq. (1) to a certain device, we first need to find out which are the cross sections which limit the current through the device, and then we have to determine both the velocity change Δv of the scattered carriers and the number N of carriers simultaneously used to test each of these cross sections or rates. Then Eq. (1) provides the spectral density of quantum 1/f cross section or rate fluctuations. These spectral densities are multiplied by the squared partial derivative of the

current, to obtain the spectral density of fractional device noise contributions from the cross sections and rates considered. After doing this with all cross sections and process rates, we add the results and bring the fine structure constant α as a common factor in front. This yields excellent agreement with the experiment[13] in a large variety of devices and physical systems. This general principle is illustrated on examples of practical applications in Sec. III, but here in Sec. II we briefly first describe the present formulation of the theory.

II.2 DERIVATION OF THE THEORY FOR N BOSONS

The simplified description of quantum 1/f noise was presented above in the elementary terms of Schrödinger's statistical catalogue model, without using second quantization. This approach is natural in view of the close connection between this new effect and diffraction which is usually treated without second quantization, in the statistical catalogue model based on the single-particle solution of the Schrödinger equation, normalized to the number of particles N. Just as the superposition of elementary phase-shifted waves allows for the simplest and most intuitive description of diffraction through a slit, the description of quantum 1/f noise in terms of interference beats between slightly frequency-shifted scattered partial waves with bremsstrahlung energy losses will always provide the simplest and most elementary quantitative derivation of the quantum 1/f effect, easily accessible even at the undergraduate level.

Below we now present the derivation of the Quantum 1/f Effect in a general form which determines the scattered current j from the observation of a sample of N outgoing particles. The minimal outgoing sample for defining particle-particle correlations in the scattered wave consists of two particles, and therefore the effect can calculated for the case of two outgoing particles. Since the general derivation also yields a 1/N factor for bosons and a factor 1/(N-1) for fermions, and since the simplifying restriction to N=2 has given rise to some misinterpretations, a presentation of the general case of N bosons or N fermions will be of interest for us as a generalization of the results derived for N=2 earlier

[#16,p.90] on the pair correlation function. In this Section we consider the case of bosons.

We start with the expression of the Heisenberg representation state $|S\rangle$ of N identical bosons of mass M emerging at an angle θ from some scattering process with various undetermined bremsstrahlung energy losses reflected in their one-particle waves $\varphi_i(\xi_i)$

$$|S\rangle = (N!)^{-1/2}\,\Pi_i \int d^3\xi_i \varphi_i(\xi_i)\psi^+(\xi_i)\,|0\rangle = \Pi_i \int d^3\xi_i \varphi_i(\xi_i)\,|S^o\rangle, \qquad (5)$$

where $\psi^+(\xi_i)$ is the field operator creating a boson with position vector ξ_i and $|0\rangle$ is the vacuum state, while $|S^o\rangle$ is the state with N bosons of position vectors ξ_i with $i = 1.....N$. All products and sums in this Section run from 1 to N, unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(\mathbf{x}_1,\mathbf{x}_2) = \psi^+(\mathbf{x}_1)\psi^+(\mathbf{x}_2)\psi(\mathbf{x}_2)\psi(\mathbf{x}_1), \qquad (6)$$

known as the operator of the pair correlation. This operator corresponds to a density autocorrelation function. The presence of two-particle coordinates in the operator O does not mean that we are considering two-particle interactions, it only means that the expectation value which we are calculating depends on the relative position of the particles. Using the well known commutation relations for boson field operators

$$\psi(\mathbf{x})\psi^+(\mathbf{y}) - \psi^+(\mathbf{y})\psi(\mathbf{x}) = \delta(\mathbf{x} - \mathbf{y}), \qquad (6'a)$$
$$\psi(\mathbf{x})\psi(\mathbf{y}) - \psi(\mathbf{y})\psi(\mathbf{x}) = 0, \qquad (6'b)$$
$$\psi^+(\mathbf{x})\psi^+(\mathbf{y}) - \psi^+(\mathbf{y})\psi^+(\mathbf{x}) = 0, \qquad (6'c)$$

we first calculate the matrix element:

$$N!\langle S^o|O|S^o\rangle$$
$$= \Sigma'_{\mu\nu}\Sigma'_{mn}\delta(\eta_\nu-\mathbf{x}_1)\delta(\eta_\mu-\mathbf{x}_2)\delta(\xi_n-\mathbf{x}_1)\delta(\xi_m-\mathbf{x}_2)\Sigma_{(i,j)}\Pi'_{ij}\delta(\eta_j-\xi_i), \qquad (7)$$

where $|S^o\rangle$ is the state with well defined particle coordinates. Here the prime excludes $\mu=\nu$ and $m=n$ in the summations and excludes $i=m$, $i=n$, $j=\mu$ and $j=\nu$ in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining N-2 values of i and j. On the basis of this result we now calculate the complete matrix element

$$\langle S|O|S\rangle = [1/N(N-1)]\ \Sigma'_{\mu\nu}\Sigma'_{mn}\int d^3\eta_\mu\int d^3\eta_\nu\int d^3\xi_m\int d^3\xi_n$$

$$\varphi_\mu^*(\eta_\mu)\varphi_\nu^*(\eta_\nu)\varphi_m(\xi_m)\varphi_n(\xi_n)\delta(\eta_\nu-\mathbf{x}_1)\delta(\eta_\mu-\mathbf{x}_2)\delta(\xi_n-\mathbf{x}_1)\delta(\xi_m-\mathbf{x}_2)$$

$$= [1/N(N-1)]\ \Sigma'_{\mu\nu}\Sigma'_{mn}\ \varphi_\mu^*(\mathbf{x}_2)\varphi_\nu^*(\mathbf{x}_1)\varphi_m(\mathbf{x}_1)\varphi_n(\mathbf{x}_2). \quad (8)$$

The one-particle states are spherical waves emerging from the scattering center located at x=0:

$$\varphi(\mathbf{x}) = (C/x)e^{iKx}\ [1 + \Sigma_{\mathbf{k}l}b(\mathbf{k},l)e^{-iqx}\ a^+_{\mathbf{k},l}]. \quad (9)$$

Here C is an amplitude factor, K the boson wave vector magnitude, $b(\mathbf{k},l)$ the bremsstrahlung amplitude for photons of wave vector $\mathbf{k}$ and polarization l, while $a^+_{\mathbf{k},l}$ is the corresponding photon creation operator, allowing the emitted photon state to be created from the vacuum if Eq. (9) is inserted into Eq. (8). The momentum magnitude loss $\hbar q=Mck/K\equiv 2\pi Mf/K$ is necessary for energy conservation in the Bremsstrahlung process. Substituting Eq, (9) into Eq. (8), we obtain

$$\langle S|O|S\rangle = |C/x|^4\{N(N-1) + 2(N-1)\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2[1+\cos q(x_1-x_2)]\}, \quad (10)$$

where we neglected a small term of higher order in $b(\mathbf{k},l)$. To perform the angular part of the summation in Eq. (10), we calculate the current expectation value of the state in Eq. (9), and compare it to the well known cross section without and with bremsstrahlung

$$j = (\hbar K/Mx^2)[1 + \Sigma_{kl}|b(k,l)|^2] = j_o[1 + \alpha A\int df/f], \quad (11)$$

where the quantum fluctuations have disappeared, $\alpha = e^2/\hbar c$ is the fine structure constant, $\alpha A = (2\alpha/3\pi)(\Delta \mathbf{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent, and where the 1/f dependence of the bremsstrahlung part displays the well-known infrared catastrophe, i.e., the emission of a logarithmically divergent number of photons in the low frequency limit. Here $\Delta \mathbf{v}$ is the velocity change $\hbar(\mathbf{K}-\mathbf{K}_0)/M$ of the scattered boson, and $f = ck/2\pi$ the photon frequency. Although the bremsstrahlung rate described by Eq. (11) is generally known, we derive it again in Sec. II.4 below. Eq. (10) thus gives

$$\langle S|O|S\rangle = |C/x|^4\{N(N-1) + 2(N-1)\alpha A\int[1+\cos q(x_1-x_2)]df/f\}, \qquad (12)$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with $df/f = dq/q$. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, (or current j, or cross section σ) fluctuations in frequency f or wave number q by dividing the coefficient of the cosine by the constant term N(N-1):

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/fN, \qquad (13)$$

where N is the number of particles or current carriers used to define the current j whose fluctuations we are studying. Quantum 1/f noise is thus a fundamental 1/N effect. The exact value of the exponent of f in Eq. (13) can be determined by including the contributions from all real and virtual multiphoton processes of any order (infrared radiative corrections), and turns out to be $\alpha A-1$, rather than -1, which is important only philosophically, since $\alpha A \ll 1$. The spectral integral is thus convergent at $f=0$.

For fermions we repeat the calculation replacing in the derivation of Eq. (10) the commutators of field operators by anticommutators, which, as we prove in detail in the next Sec. II.3, finally yields in the same way

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1), \qquad (14)$$

which causes no difficulties, since $N \geq 2$ for particle correlations to be defined, and which is practically the same as Eq. (13), since usually $N>>1$. Eqs. (13) and (14) suggest a new notion of physical cross sections and process rates which contain 1/f noise, and express a fundamental law of physics, important in most high-technology applications[13].

We conclude that the conventional quantum 1/f effect can be explained in terms of interference beats between the part of the outgoing DeBroglie waves scattered without bremsstrahlung energy losses above the detection limit (given in turn by the reciprocal duration T of the 1/f noise measurement) on one hand, and the various parts scattered with bremsstrahlung energy losses; but there is more to it than that: exchange between identical particles is also important. This, of course, is just one way to describe the reaction of the emitted bremsstrahlung back on the scattered current. This reaction thus reveals itself as the cause of the quantum 1/f effect, and implies that the effect can not be obtained with the independent boson model. The effect, just like the classical turbulence-generated 1/f noise[14], is a result of the scale-invariant nonlinearity of the equations of motion describing the coupled system of matter and field. Ultimately, therefore, this nonlinearity is the source of the 1/f spectrum in both the classical and quantum form of the author's theory. We can say that the quantum 1/f effect is an infrared divergence phenomenon, this divergence being the result of the same nonlinearity. The new effect is, in fact, the first time-dependent infrared radiative correction. Finally, it is also deterministic in the sense of a well determined wave function, once the initial phases γ of all field oscillators are given. In quantum mechanical correspondence with its classical turbulence analog, the new effect is therefore a quantum manifestation of classical chaos which we can take as the definition of a certain type of <u>quantum chaos</u>.

II.3 DERIVATION FOR THE CASE OF N FERMIONS

In the case of fermions the calculation is similar, except for the use of anticommutators for the fermion field operators. In order to emphasize the independence of our results on the representation used, and to show directly how the calculations presented in this Section can be performed without second quantization, we give here the direct calculation in terms of a Slater determinant for the state of N scattered fermions

$$\Psi_{i_1,\ldots i_N}(\mathbf{r}_1\ldots\mathbf{r}_n) = (N!)^{-1/2}\begin{vmatrix} \phi_{i_1}(\mathbf{r}_1) & \phi_{i_1}(\mathbf{r}_2)\ldots\ldots\phi_{i_1}(\mathbf{r}_n) \\ \phi_{i_2}(\mathbf{r}_1) & \phi_{i_2}(\mathbf{r}_2)\ldots\ldots\phi_{i_2}(\mathbf{r}_n) \\ \cdots\cdots & \cdots\cdots\cdots\cdots \\ . & \\ . & \\ \phi_{i_N}(\mathbf{r}_1) & \phi_{i_N}(\mathbf{r}_2)\ldots\ldots\phi_{i_N}(\mathbf{r}_n) \end{vmatrix} \tag{15}$$

Here $\mathbf{r}$ combines the position vector $\mathbf{x}$ and the spin variable s for each of the particles. The pair-correlation function is obtained by integrating with respect to the coordinates of all but two of the fermions

$$A(\mathbf{r}_1,\mathbf{r}_2) = \int d^3r_3\ldots d^3r_n<|\Psi_{i1\ldots iN}(\mathbf{r}_1\ldots\mathbf{r}_n)|^2>. \tag{16}$$

Here the integrals also include summations over the spins; the expectation value is with respect to the phases present in the bremsstrahlung parts of the wave functions. If the emitted photons are included in the final state, the expectation value is also taken on the vacuum background of the photons. Assuming orthonormality of the functions $\phi_{i1}\ldots\phi_{iN}$, we obtain

$$A(\mathbf{r}_1,\mathbf{r}_2) = [1/N(N-1)]\sum_{m;n=1}^{N}<|\phi_m(\mathbf{r}_1)\phi_n(\mathbf{r}_2) - \phi_m(\mathbf{r}_2)\phi_n(\mathbf{r}_1)|^2>. \tag{17}$$

To display the spin variables explicitly, we write $\phi_m(\mathbf{r}) = \chi_m(\mathbf{x})|s>$ and get

$$A_{ss'}(x_1,x_2) = [1/N(N-1)] \sum_{m;n=1}^{N/2} <[\chi_m^*(x_1)<s|\chi_n^*(x_2)<s'| -$$

$$\chi_m^*(x_2)<s'|\chi_n^*(x_1)<s|]\,[\chi_m(x_1)s>|\chi_n(x_2)s'>| - \chi_m(x_2)s'>|\chi_n(x_1)s>|]>$$

$$= [1/N(N-1)] \sum_{m;n=1}^{N/2} <[|\chi_m(x_1)|^2|\chi_n(x_2)|^2 + |\chi_m(x_2)|^2|\chi_n(x_1)|^2$$

$$- \chi_m^*(x_1)\chi_m(x_2)|<s|s'>|^2\chi_n^*(x_2)\chi_n(x_1) - (x_1\Leftrightarrow x_2)]> \qquad (18)$$

Here the symbol $(x_1\Leftrightarrow x_2)$ designates the immediately preceeding term with x_1 and x_2 interchanged. Considering all spin orientations, we obtain

$$A(x_1,x_2) = A_{\uparrow\uparrow} + A_{\uparrow\downarrow} = [1/N(N-1)] \sum_{m;n=1}^{N/2} <[4|\chi_m|^2|\chi_n|^2$$

$$- \chi_m^*(x_1)\chi_n(x_2)\chi_m^*(x_2)\chi_n(x_1) - (x_1\Leftrightarrow x_2)]> \qquad \text{(form 1)}$$

$$= [(|C|^4)/x_1^2x_2^2N(N-1)]\Big\{N^2[1+\Sigma_{k,l}|b(k,l)|^2]^2$$

$$- 2\sum_{m=1}^{N/2} \exp[iK_m(x_1-x_2)][1 + \Sigma_{k,l}|b_m(k,l)|^2\exp[-iq_m(x_1-x_2)]$$

$$\times \sum_{n=1}^{N/2} \exp[-iK_n(x_1-x_2)][1 + \Sigma_{k',l'}|b_n(k',l')|^2\exp[iq_n(x_1-x_2)]\Big\} \qquad \text{(form 2)}$$

$$\approx [(C^4)/x_1^2x_2^2(N-1)]\{N[1 + \Sigma_{k,l}|b(k,l)|^2]^2$$

$$- (2/N)\sum_{n=1}^{N/2} [1 + 2\Sigma_{k,l}|b_n(k,l)|^2\cos q_n(x_1-x_2)$$

$$+ \Sigma_{k,l;k',l'}|b_n(k,l)|^2|b_n(k',l')|^2\cos(q_n-q'_n)(x_1-x_2)]\} \qquad \text{(form 3)}$$

$$\approx[(C^4)/x_1^2x_2^2(N-1)]\{N[1+\Sigma_{k,l}|b(k,l)|^2]^2-1-2\Sigma|b(k,l)|^2\cos q(x_1-x_2)]\} \qquad (19)$$

This form of the pair-correlation function includes the 1/N factor which multiplies the variable (noise) part. The crucial point in the derivation of the 1/N factor was an elimination of the rapidly oscillating terms $\exp i(K_n-K_m)(x_1-x_2)$ with $K_n\neq K_m$ present in the second form of Eq. (19) above, an elimination indicated through the approximation sign connecting the second form to the third form above. Indeed, since K_m differs from K_n by much more then the momentum change corresponding to the emission of an infraquantum, these terms will have a very fast oscillation, and will

not yield any low frequency noise. Since they are also small in magnitude, they are negligible. This provides the important reduction of the noise term by a factor N. In the last form of Eq. (18) we note that the first two terms are constant and large, and do not yield any rapid oscillations which would justify elimination of any cross terms with $K_n \neq K_m$. Thus, in Eq. (19) the constant part of the pair correlation is not reduced in magnitude, i.e., only the noise part is affected.

In Eq. (19) we have used again the form of the single-particle wave functions given by Eq. (9) and derived below in Eq. (41), with independent sets of phases present in the bremsstrahlung energy loss parts of each particle. Had we used a random time-shift in each of the single-particle wave functions, i.e. just a random initial time constant, or, equivalently, a random space-shift-constant, all results would have been exactly the same. In fact, the random shift should better describe the initial Poisson distribution of the incoming particles which are scattered. As we will see at the end of Sec. II.4 below, the random phase set in the bremsstrahlung energy loss parts is the set of random initial phases of the electromagnetic field oscillators. Therefore it should come in the same way for all particles. However, the random shift will eliminate expectations of the cross products $b_m^*(k,l)b_n(k,l)$ with $m \neq n$ just as the sets of random phases used by us did; if ρ is for instance a random space shift, these cross terms yield contributions to the pair-correlation function of the form $\langle |b(k,l)|^2 \cos q(x_1 - x_2 + \rho)\rangle = 0$, where the average is with respect to ρ. We present this observation here as an afterthought, because the sets of random phases generated by a shift in time or space will appear to be random, but will still contain some correlations. The point we make here is that these correlations have no effect on our calculations, so we can continue to use random phases for our purpose. There may be, perhaps, some differences in higher-order correlations which we do not consider here.

In the last form of Eq. (19) we have neglected the higher-order term. Performing the sum in the same way as in Eqs. (10)-(12), or as we show in Sec.II.4, we can write the pair-correlation function for fermions in the form

$$A(x_1,x_2) = [(|C|^4)/x_1^2x_2^2]\{1 \ -2(N-1)^{-1}\Sigma_{k,l}|b(k,l)|^2[N-\cos q(x_1-x_2)]\} \quad (20)$$

Dividing again the variable part by the constant term, and neglecting small constant terms, we finally obtain for the fractional spectral density of the fermion current and cross section fluctuations

$$S_j(k)dk/j^2 = 2\alpha Adk/k(N-1) = S_j(f)df/j^2 = \underline{2\alpha Adf/f(N-1)}. \quad (21)$$

II.4 DERIVATION OF THE SINGLE-PARTICLE WAVE FUNCTIONS

The derivation of the Schrödinger field used in II.1 - II.3 will be performed in this Section with the help of the Green's function method similar to the method used in a earlier calculation by Kroll and Watson[15], extended by the author to the case of interaction with all electromagnetic modes of the universe[7].

It is most convenient to describe the electromagnetic field in terms of plane waves. The vector potential is taken in the radiation gauge as

$$\mathbf{A}(\mathbf{r},t) = \Sigma_{\mathbf{k},l}(h^2c/L^3\omega_k)^{1/2}\mathbf{u}_{\mathbf{k},l}[a_{\mathbf{k},l}(t)e^{i\mathbf{kr}} + a^*_{\mathbf{k},l}(t)e^{-i\mathbf{kr}}]. \quad (22)$$

The polarization vectors $u_{\mathbf{k},1}$ and $u_{\mathbf{k},2}$ are mutually orthogonal unit vectors perpendicular to $\mathbf{k}$.

The Schrödinger equation for an electron moving in a vector potential $\mathbf{A}$ and scattering potential V is

$$(1/2m)[-i\hbar\nabla - e\mathbf{A}/c]^2\psi + V\psi = i\hbar\dot{\psi}. \quad (23)$$

A dot has been used to indicate the time derivative. The electromagnetic field is treated as a classical field at this point. In order to eliminate the $\mathbf{A}^2$ term from Eq. (23), we write

$$\Psi = \exp[(-i/\hbar)\int^t(e^2/2mc^2)\mathbf{A}^2dt']\Phi. \quad (24)$$

Thus, Eq. (4.2) is reduced to

$$[(-\hbar^2/2m)\nabla^2 + (ie\hbar/mc)\mathbf{A}\cdot\nabla + V]\Phi = i\hbar\dot{\Phi}. \quad (25)$$

It is convenient to consider first the influence of a single electromagnetic mode, i.e. a single term from Eq. (22). Therefore, we take $\mathbf{A} = \mathbf{a}\cos(\omega t+\gamma)$, where γ is an initial phase constant, and we treat $V\Phi$ as a perturbation source term. The solution for Eq. (25) is an incoming plane wave plus scattered waves, given by the integral equation

$$\Phi_{\mathbf{k}o}(\mathbf{r},t) = \phi_{\mathbf{k}o} - \int d^3x'\int^t dt' G V(\mathbf{r}')\Phi_{\mathbf{k}o}(\mathbf{r}',t'). \tag{26}$$

Here $\phi_{\mathbf{k}o}$ is the solution of the homogeneous equation, i.e. with $V = 0$, and can be written in the form

$$\phi_{\mathbf{k}o} = \exp[i\mathbf{k}_o\mathbf{r}]\exp[-(i\hbar/2m)\int^t(k^2{}_o - 2e\mathbf{k}_o\cdot\mathbf{A}/c\hbar)dt]. \tag{27}$$

G is the Green's function which satisfies the equation

$$[(-\hbar^2/2m)\nabla^2 + (ie\hbar\mathbf{A}\cdot\nabla/mc) - i\hbar\partial/\partial t]G = \delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \tag{28}$$

Given $\mathbf{A} = \mathbf{a}\cos(\omega t + \gamma)$, G can be found to be

$$G = [i/(2\pi)^3\hbar]\int d^3k\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\exp\{-(i\hbar/2m)[k^2t - 2e\mathbf{k}\cdot\mathbf{a}\sin(\omega t + \gamma)/\hbar c\omega]\}$$
$$\times \exp[(i\hbar/2m)(k^2t' - 2e\mathbf{k}\cdot\mathbf{a}\sin(\omega t + \gamma)/\hbar c\omega] \tag{29}$$

In the first Born approximation we set $\Phi_{\mathbf{k}}(\mathbf{r}',t') = \phi_{\mathbf{k}}(\mathbf{r}',t')$ in the integral present in Eq. (26) and obtain for the scattered wave

$$\Psi_s = [i/(2\pi)^3\hbar]\int d^3x'\int^t dt' V(\mathbf{r}')\int d^3k\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}\exp\{-(i\hbar/2m)$$
$$[k^2t - 2e\mathbf{k}\cdot\mathbf{a}\sin(\omega t + \gamma)/\hbar c\omega]\} \times \exp[(i\hbar/2m)(K^2 - k_o{}^2)t']$$
$$\times \{\exp[ie(\mathbf{k}_o - \mathbf{k})\mathbf{a}\sin(\omega t' + \gamma)/2mc\omega]e^{i\mathbf{k}_o\mathbf{r}'}\}. \tag{30}$$

Using the relation

$$e^{i\beta\sin(\omega t'+\gamma)} = \sum_{n=-\infty}^{\infty} J_n(\beta)e^{in(\omega t'+\gamma)}, \tag{31}$$

where $J_n(\beta)$ is the n^{th} order Bessel function, we expand the expression contained in curly brackets in Fourier series. Then Eq. (26) takes the form

$$\Phi_{\mathbf{k}}(\mathbf{r},t) - \phi_{\mathbf{k}}(\mathbf{r},t) = [-i/(2\pi)^3\hbar]\int d^3x'\int^t dt' V(\mathbf{r}')\int d^3k\, e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}')}$$
$$\times \exp\{-(i\hbar/2m)[k^2t - 2e\mathbf{k}\mathbf{a}\sin(\omega t+\gamma)/\hbar c\omega]\}$$
$$\times \exp[(i\hbar/2m)(k^2 - k^2)t'][\sum_{n=-\infty}^{\infty} J_n(\beta)e^{in(\omega t'+\gamma)}]e^{i\mathbf{k}\,\mathbf{r}'}. \quad (32)$$

After performing the integration over t' we use a contour integration method for $\mathbf{k}$. Then Eq. (32) is reduced to

$$\Phi_{\mathbf{k}}(\mathbf{r},t) - \phi_{\mathbf{k}}(\mathbf{r},t) = [-m/(2\pi)\hbar^2]\sum_{n=-\infty}^{\infty} J_n(\beta)e^{in\gamma}[e^{ik(n)r}/r]$$

$$\exp\{i\hbar[k^2(n)t-2e\mathbf{k}(n)\mathbf{a}\sin(\omega t+\gamma)/\hbar c\omega]/2m\}\int d^3x' e^{i\mathbf{k}(n)\mathbf{r}'}V(\mathbf{r}')e^{i\mathbf{k}\mathbf{r}'}, \quad (33)$$

where

$$\beta = -e(\mathbf{k}_n - \mathbf{k}_o)\cdot\mathbf{a}/mc\omega = -e\mathbf{Q}\cdot\mathbf{a}/mc\hbar\omega, \quad (34)$$

and Q is the momentum transfer. In Eq. (33) $\mathbf{k}(n)$ is defined by

$$(\hbar k(n))^2/2m = (\hbar k_o)^2/2m - n\hbar\omega;\ \mathbf{k}_n = \mathbf{k}(n) = k(n)\mathbf{r}/r. \quad (35)$$

The total scattered wave can be written as

$$\psi_s = [-m/(2\pi)\hbar^2 r]\sum_{n=-\infty}^{\infty} e^{ik(n)r}\exp\{-i\hbar[k^2(n)t-2e\mathbf{k}(n)\cdot\mathbf{a}\sin(\omega t+\gamma)/\hbar c\omega]/2m\}$$
$$\times V_{\mathbf{k}(n),\mathbf{k}}\, J_n(\beta)e^{in\gamma}, \quad (36)$$

where $V_{\mathbf{k}(n),\mathbf{k}} = \int e^{-i\mathbf{k}(n)\cdot\mathbf{r}'}V(\mathbf{r}')e^{i\mathbf{k}_o\cdot\mathbf{r}'}d^3x'$ is the scattering matrix element calculated without consideration of the interaction with the electromagnetic field oscillators.

So far the electromagnetic field has not been quantized and was considered as a classical field. We are interested in the corresponding expression of the scattered single-particle wave function when the electromagnetic oscillators are quantized. Therefore we first linearize Eq. (36) with respect to the electromagnetic potential wherever a dependence on ξ is present:

$$\psi_s = [-m/(2\pi)\hbar^2 r]V_{\mathbf{K},\mathbf{k}}\, e^{iKr-iEt/\hbar}\exp[ie\mathbf{K}\cdot\mathbf{a}\sin(\omega t+\gamma)/cm\omega]$$
$$\{1 + e^{i(\omega t-qr+\gamma)}\beta/2 - e^{-i(\omega t-qr+\gamma)}\beta/2\}$$
$$= [-m/(2\pi)\hbar^2 r]V_{\mathbf{K},\mathbf{k}}\, e^{iKr-iEt/\hbar}\exp[1 + ei\mathbf{K}\cdot\mathbf{a}\sin(\omega t+\gamma)/cm\omega]$$
$$\{1 + e^{i(\omega t-qr+\gamma)}e\mathbf{Q}\cdot\mathbf{a}/2mc\hbar\omega - e^{-i(\omega t-qr+\gamma)}e\mathbf{Q}\cdot\mathbf{a}/2mc\hbar\omega\}. \quad (37)$$

Since $k = (v/c)q<<q$, the $\mathbf{r}$ dependence of the elctromagnetic potential can be neglected, as it is in exp $[ie\mathbf{K}\cdot\mathbf{a}\sin(\omega t+\gamma)/cm\omega]$.

Here we have introduced the notations $E = (\hbar K)^2/2m$ and $\mathbf{K} = \mathbf{k}(0)$. In the last form we will neglect the term with $\sin(\omega t + \gamma)$ because it corresponds to a coherent quantum 1/f noise contribution which has been considered elsewhere before[11,12], and because this term can be considered constant of negligible magnitude in the calculation of equal time spatial correlations. We also neglected the small difference between $\mathbf{k}(1) = \mathbf{K} - \omega\mathbf{K}/vK$ and $\mathbf{K}$ in β and $V_{\mathbf{k}(1),\mathbf{k}}$. We conclude that the quantization of the electromagnetic field transforms ψ_s into an operator, just because $a_{\mathbf{k},l}$ is an operator:

$$\phi(\mathbf{r},t) = (C/r)e^{iKr-iEt/\hbar}\{1 - \Sigma_{\mathbf{k},l}b^*(\mathbf{k},l)\exp i(-\omega t + qr - \gamma_l)a_{\mathbf{k},l} + \Sigma_{\mathbf{k},l}b(\mathbf{k},l)\exp i(\omega t - qr + \gamma_l)a^+_{\mathbf{k},l}\}. \tag{38}$$

Here we have introduced $b(\mathbf{k},l) = (1/2)\beta$ and the constant C which designates the factor in front, and a sum which includes all electromagnetic modes with annihilation operators $a_{\mathbf{k},l}$. In the space of the electron states $\phi(\mathbf{r},t)$ is just a single-particle wave function. We have denoted by q the small decrease in the particle momentum required by Eq. (35). We have $q = (\partial K/\partial E)\hbar\omega = ck/v = \omega/v$, with

$$(L/2\pi)^3\cdot 4\pi\cdot\Sigma_l<|b(\mathbf{k},l)|^2>k^2dk = (e^2Q^2a^2/4m^2c^2\hbar^2\omega^2)\cdot 2k^2d\omega/3c = \alpha A d\omega/\omega, \tag{39}$$

where $\alpha = e^2/4\pi\hbar c = 1/137$, $A = (2Q^2/3\pi m^2c^2)$, and where $< >$ is an angular average. We have considered the spontaneous emission caused by vacuum fluctuations only, yielding

$$<a_{\mathbf{k},l}> = c(2\hbar/\omega L^3)^{1/2}. \tag{40}$$

The annihilation part is included in Eq. (38), but does not contribute to the quantum 1/f noise on the background of the electromagnetic vacuum, so we can simply write

$$\varphi(\mathbf{x}) = (C/r)e^{iKr-iEt/\hbar}\,[1 + \Sigma_{\mathbf{k}l} b(\mathbf{k},l)e^{i(\omega t - qr + \gamma_i)}\, a^{+}_{\mathbf{k},l}], \qquad (41)$$

which is identical with Eq. (9) for t=0, which in turn corresponds to the Heisenberg representation state vector. If the calculation is performed on the thermal radiation background, however, we get from the $a^{+}_{\mathbf{k},l}$ terms a white noise contribution added to the quantum 1/f noise; the latter, however, remains the same[4]. Here we have performed the transition from just one electromagnetic mode to the general case with all electromagnetic modes adhoc, but in our paper[7] this transition was presented in detail.

In Eq. (38) we notice the presence of the random phases γ_i which where introduced as initial phases of the electromagnetic oscillators, and which are independent for each electromagnetic mode i (of given polarization l and wave vector **k**) of the universe. Since the various scattered particles are independent of each other and of the electromagnetic modes, we have to consider the set of random phases γ_i different and independent in the wave function of each particle.

II.5 DERIVATION OF THE PAIR-CORRELATION IN TIME AND SPACE

Working in the Heisenberg representation as above, we can generalize the derivation presented above by including two different times in the operator of the pair-correlation, although this requires more calculation. To simplify the integrals, we consider again the state of two outgoing fermions very far from the place where they have independently suffered the same interaction, so that the outgoing spherical waves can be approximated by plane waves. Starting therefore with plane waves similar to the spherical waves used in Eqs. (38) and (9), with $\mathbf{q} = q\mathbf{K}/K = ck\mathbf{K}/Kv$ and $\mathbf{q}' = ck'\mathbf{K}/Kv$, we obtain again the pair-correlation function in the form

$$\begin{aligned} A(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) = {} & 1/2\int d^3\xi\, d^3\eta\, d^3\xi' d^3\eta' \\ & \{\exp(-i\mathbf{K}\eta') + \Sigma_{\mathbf{k}',s'}\beta^*(\mathbf{k}',s')\exp[-i(\mathbf{K}-\mathbf{q}')\eta']a_{\mathbf{k}',s'}\} \\ & \{\exp(-i\mathbf{K}\xi') + \Sigma_{\mathbf{k},s}b^*(\mathbf{k},s)\exp[-i(\mathbf{K}-\mathbf{q})\xi']a_{\mathbf{k},s}\} \end{aligned}$$

$$\{\exp(iK\xi) + \Sigma_{\mathbf{k},s} b(\mathbf{k},s)\exp[i(\mathbf{K}-\mathbf{q})\xi]a^+_{\mathbf{k},s}\}$$
$$\{\exp(iK\eta) + \Sigma_{\mathbf{k}',s'}\beta(\mathbf{k}',s')\exp[i(\mathbf{K}-\mathbf{q}')\eta]a^+_{\mathbf{k}',s'}\}$$
$$(1/4)\Sigma_{ss'}<S^o{}_{ss'}|O_{\uparrow\uparrow} + O_{\uparrow\downarrow} + O_{\downarrow\uparrow} + O_{\downarrow\downarrow}|x_1,t_1;\ x_2,t_2\ |S^o{}_{ss'}>. \tag{42}$$

The operator of the pair-correlation function now contains two consecutive times. The creation and annihilation operators for particles obey *anticommutation* relations similar to the commutation relations in Eqs. (6'a)-(6'c). Using these operators, the field operators can be expanded in terms of plane waves,

$$\psi_s(\mathbf{r},t) = V^{-1/2}\Sigma_{\mathbf{p}}\ \exp i(\mathbf{p}\mathbf{r}-\omega t)c_{\mathbf{p}s},$$
$$\psi^+{}_s(\mathbf{r},t) = V^{-1/2}\Sigma_{\mathbf{p}}\ \exp i(-\mathbf{p}\mathbf{r}+\omega t)c^+{}_{\mathbf{p}s} \tag{43}$$

which also contain their time dependence, as needed for the operator of the pair correlation function in the Heisenberg representation. This operator has an expectation value which can be written, for spin up only, in the form

$$<S^o\ |O_{\uparrow\uparrow}(\mathbf{x}_1,\ t_1,\ \mathbf{x}_2,\ t_2)|S^o>$$
$$= <0|\psi(\eta')\psi(\xi')\psi^+(\mathbf{x}_1,t_1)\psi^+(\mathbf{x}_2,t_2)\psi(\mathbf{x}_2,t_2)\psi(\mathbf{x}_1,t_1)\psi^+(\xi)\psi^+(\eta)|0>$$
$$= (1/V^4)\sum_{\mathbf{mnm'n'}}\ \sum_{\mu\nu\mu'\nu'} \exp i[\mathbf{m}'\eta' + \mathbf{n}'\xi' - \mu'\mathbf{x}_1 + \omega(\mu')t_1 - \nu'\mathbf{x}_2 + \omega(\nu')t_2$$
$$+\mu\mathbf{x}_2 - \omega(\mu)t_2 + \nu\mathbf{x}_1 - \omega(\nu)t_1 - \mathbf{m}\eta - \mathbf{n}\xi]<0|c_{\mathbf{m}'}c_{\mathbf{n}'}c^+{}_{\nu'}c^+{}_{\mu'}c_\mu c_\nu c^+{}_{\mathbf{m}}c^+{}_{\mathbf{n}}|0>$$

$$= (M/h)^6 t_1{}^{-3}t_2{}^{-3}\exp\{(iM/2\hbar)[(\mathbf{x}_1-\xi)^2/t_1 + (\mathbf{x}_2-\eta)^2/t_2$$
$$-(\mathbf{x}_1-\eta')^2/t_1 - (\mathbf{x}_2-\xi')^2/t_2]\}$$
$$+(M/h)^6 t_1{}^{-3}t_2{}^{-3}\exp\{(iM/2\hbar)[(\mathbf{x}_1-\eta)^2/t_1 + (\mathbf{x}_2-\xi)^2/t_2$$
$$-(\mathbf{x}_1-\xi')^2/t_1 - (\mathbf{x}_2-\eta')^2/t_2]\}$$
$$-(M/h)^6 t_1{}^{-3}t_2{}^{-3}\exp\{(iM/2\hbar)[(\mathbf{x}_1-\xi)^2/t_1 + (\mathbf{x}_2-\eta)^2/t_2$$
$$-(\mathbf{x}_1-\xi')^2/t_1 - (\mathbf{x}_2-\eta')^2/t_2]\}$$
$$-(M/h)^6 t_1{}^{-3}t_2{}^{-3}\exp\{(iM/2\hbar)[(\mathbf{x}_1-\eta)^2/t_1 + (\mathbf{x}_2-\xi)^2/t_2$$
$$-(\mathbf{x}_1-\eta')^2/t_1 - (\mathbf{x}_2-\xi')^2/t_2]\}. \tag{44}$$

Substituting into Eq. (42), and integrating with respect to ξ,η,ξ' and η', we obtain for the part with spin up only

$$
\begin{aligned}
&8A_{\uparrow\uparrow}(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) \\
&= \langle 0\{1 + \Sigma_{\mathbf{k}'}\beta^*(\mathbf{k}')\exp i[\mathbf{q}'\mathbf{x}_1 - \hbar q'(K - q'/2)t_1/M]a_{\mathbf{k}'}\} \\
&\quad \{1 + \Sigma_{\mathbf{k}} b^*(\mathbf{k})\exp i[\mathbf{q}\mathbf{x}_2 - \hbar q(K - q/2)t_2/M]a_{\mathbf{k}}\} \\
&\quad \{1 + \Sigma_{\mathbf{k}} b(\mathbf{k})\exp i[\mathbf{q}\mathbf{x}_1 + \hbar q(K - q/2)t_1/M]a^+_{\mathbf{k}}\} \\
&\quad \{1 + \Sigma_{\mathbf{k}'}\beta(\mathbf{k}')\exp i[\mathbf{q}'\mathbf{x}_2 + \hbar q'(K - q'/2)t_2/M]a^+_{\mathbf{k}'}\}0\rangle \\
&= 1 + \Sigma_{\mathbf{k}'}|b(\mathbf{k}')|^2\exp i[\mathbf{q}'(\mathbf{x}_1 - \mathbf{x}_2) - \hbar q'(K - q'/2)(t_1 - t_2)/M] \\
&\quad + \Sigma_{\mathbf{k}}|b(\mathbf{k})|^2\exp i[\mathbf{q}(\mathbf{x}_2 - \mathbf{x}_1) - \hbar q(K - q/2)(t_2 - t_1)/M] \\
&\quad + \Sigma_{\mathbf{k},\mathbf{k}'}|b(\mathbf{k})b(\mathbf{k}')|^2\exp i[(\mathbf{q} - \mathbf{q}')(\mathbf{x}_2 - \mathbf{x}_1) - \hbar K(q - q')(t_2 - t_1)/M \\
&\quad + \hbar(q^2 - q'^2)(t_2 - t_1)/2M]. \qquad (45)
\end{aligned}
$$

In a similar way the expectation values corresponding to the $\uparrow\downarrow$ spin orientations and the two identical terms with all spins reversed are calculated and integrated with respect to ξ, η, ξ' and η'. Putting together the terms for all spin orientations as before, we finally obtain the desired pair correlation function in the form

$$
\begin{aligned}
&A(\mathbf{x}_1, t_1; \mathbf{x}_2, t_2) = 1/2 + \Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2\{2 - \cos\mathbf{q}[\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{v}'(t_1 - t_2)]\} \\
&+ \Sigma_{\mathbf{k}\mathbf{k}'ll'}|b(\mathbf{k},l)|^2|b(\mathbf{k}',l)|^2\{1 - (1/2)\cos(\mathbf{q}-\mathbf{q}')[\mathbf{x}_1 - \mathbf{x}_2 - \mathbf{v}''(t_1 - t_2)]\}; \qquad (46) \\
&\mathbf{v}' = \mathbf{v}(1 - q/2K) = \mathbf{v}(1 - \varepsilon/4E) \approx \mathbf{v};\ \mathbf{v}'' = \mathbf{v}[1 - (q+q')/2K] = \mathbf{v}[1 - (\varepsilon+\varepsilon')/4E] \approx \mathbf{v}.
\end{aligned}
$$

The approximations $\mathbf{v}' \approx \mathbf{v}$ and $\mathbf{v}'' \approx \mathbf{v}$ are justified because the soft photon energy $\varepsilon = 4\ 10^{-15}$eV for 1Hz is negligible compared to the energy E of the particles which may be of the order of 1eV. This result directly proves the validity of our translation of wave-number spectra into frequency spectra in Sec. II.2 and II.3. The extension to the case of bosons is trivial.

II.6. QUANTUM 1/f CROSS-CORRELATIONS AND SPECTRA

The calculation of quantum 1/f noise in various kinetical coefficients governing transport in condensed matter, requires a knowledge of the cross-correlation of the outgoing current densities scattered by any process into different directions with wave vectors $\mathbf{K}'$ and $\mathbf{K}''$ when the incoming particles had wave vectors $\mathbf{K}_1$ and $\mathbf{K}_2$. The particles are assumed to be identical, of mass M, with wave functions which overlap

somewhere. As we have seen in the preceding Secs. II.2-3 on the general derivation of quantum 1/f noise in physical cross sections, the time correlations and frequency spectra can be readily obtained from the corresponding spatial correlations along the outgoing beams in the direction of the detector which is now replaced by two detectors in the directions of **K'** and **K"** respectively. This is what we called translation from wave-numbers to frequencies at the end of the last Section. Working again in the Heisenberg representation, we limit ourselves to the simplest case of two particles described by the outgoing spherical waves

$$\varphi(x) = (C/x)\exp(iK_1x)[1 + \Sigma_{\mathbf{k},l}b(\mathbf{k},l)\exp(-iq_1x)a^+_{\mathbf{k},l}]|0> \qquad (47)$$

$$\chi(x) = (C/x)\exp(iK_2x)[1 + \Sigma_{\mathbf{k},l}\beta(\mathbf{k},l)\exp(-iq_2x)a^+_{\mathbf{k},l}]|0> \qquad (48)$$

where now b corresponds to the momentum change $\mathbf{K}'-\mathbf{K}_1$, while β has an independent phase and corresponds to the momentum change $\mathbf{K}"-\mathbf{K}_2$ of the second particle. As in the preceding Sec. II.2, <u>for bosons</u> we obtain the current density autocorrelation function

$$A(\mathbf{K}_1,\mathbf{K}_2,\mathbf{K}',\mathbf{K}";x_1x_2) = <S|O|S> = (K'K"/2M)<0|\chi(x_1)\varphi(x_2)|^2$$

$$+\chi^*(x_1)\varphi^*(x_2)\chi(x_2)\varphi(x_1) +\chi^*(x_2)\varphi^*(x_1)\chi(x_1)\varphi(x_2) +|\chi(x_2)\varphi(x_1)|^2|0>$$

$$= (K'K"/M)\{[1 + \Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2][1 + \Sigma_{\mathbf{k},l}|\beta(\mathbf{k},l)|^2]$$

$$+(1/2)\exp[iK'(x_2-x_1)-iK"(x_2-x_1)][1 + \Sigma_{\mathbf{k},l}|\beta(\mathbf{k},l)|^2\exp{-iq(x_2-x_1)}]$$

$$[1 + \Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2\exp{iq(x_2-x_1)}] + [x_1 \leftrightarrow x_2]\}$$

$$= (K'K"/M)\{[1 +\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2][1 +\Sigma_{\mathbf{k},l}|\beta(\mathbf{k},l)|^2] + \cos(K'-K")(x_2-x_1)$$

$$+ \Sigma_{\mathbf{k},l}|\beta(\mathbf{k},l)|^2\cos[(K'-K"-q)(x_2-x_1) +\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2\cos[(K'-K"+q)(x_2-x_1)$$

$$+ \Sigma_{\mathbf{k},l}\Sigma_{\mathbf{k}',l'}|\beta(\mathbf{k},l)|^2|b(\mathbf{k}',l')|^2\cos[(K'-K"+q'-q)(x_2-x_1)]\}. \qquad (49)$$

Except for K'=K" all terms which are not constant are rapidly oscillating and do not yield low frequency noise contributions; as usual, only practically unobservable high $1/\Delta f$ noise terms are present in this case in the quantum fluctuations. For K'=K", however, we get low frequency noise down to arbitrarily low frequencies, because the momentum magnitude changes $q=kMc/\hbar K$ and $q'=k'Mc/\hbar K$ caused by the soft photon emission are arbitrarily small. We obtain the spatial quantum 1/f current density cross-correlation

$$A(\mathbf{K}_1,\mathbf{K}_2,K',K";x_1x_2) = (\hbar^2K'K"/M)\{[1+\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2][1+\Sigma_{\mathbf{k},l}|\beta(\mathbf{k},l)|^2]$$

$$+ \{1 + \Sigma_{\mathbf{k},l}[|b(\mathbf{k},l)|^2 + |\beta(\mathbf{k},l)|^2]\cos q(x_2-x_1)\}\delta_{K',K"}$$

$$+ \Sigma_{\mathbf{k},l}\Sigma_{\mathbf{k}',l'}|\beta(\mathbf{k},l)|^2|b(\mathbf{k}',l')|^2\cos[(q'-q)(x_2-x_1)]\delta_{K',K"}\}. \qquad (50)$$

Note that the restriction indicated by the Kronecker symbol refers only to the magnitude of the outgoing wave vectors, or to the kinetic energies of the outgoing particles, and that their outgoing and incoming directions can be different. Therefore, only particles of the same energy have quantum 1/f noise cross-correlations, and particle groups of different energy will yield independent quantum 1/f noise contributions, a result which was anticipated empirically by Kleinpenning in semiconductors. Performing the angular integrations contained in the softaron lattice sums of Eq. (50), as indicated in Eq. (39), and using the translation relation $dk/k = df/f$, with $f=ck/2\pi$, we obtain

$$A(\mathbf{K}_1,\mathbf{K}_2,K',K";x_1x_2) = (\hbar^2K'K"/M)\{[1 +\alpha A_1\int df/f][1 +\alpha A_2\int df/f]$$

$$+ [1 + \alpha\int(A_1 + A_2)\cos\, q(x_2-x_1)\ (df/f)]\delta_{K',K"}$$

$$+ \alpha^2A_1A_2\int(df/f)\int(df'/f')\cos[(q'-q)(x_2-x_1)]\delta_{K',K"}\}. \qquad (51)$$

Here $A_1 = 2\hbar^2(\mathbf{K}'-\mathbf{K}_1)^2/3\pi M^2c^2$ and $A_2 = 2\hbar^2(\mathbf{K}"-\mathbf{K}_2)^2/3\pi M^2c^2$. We may call this approximation the independent bremsstrahlung model. Neglecting the

last term which is a noise of noise contribution, applying the Wiener-Khintchine theorem, and using again the relation dk/k = df/f, to connect wave number spectra to frequency spectra, we obtain the cross-spectral densities of the rate fluctuations $\Delta w(K_1,K')$ from K_1 to K', and $\Delta w(K_2,K'')$ from K_2 to K''

$$S_{\Delta w}(K_1,K_2,K',K'') = (\alpha/2f)(A_1+A_2)<w(K_1,K')><w(K_2,K'')>\delta_{K',K''}(2/N) \qquad (52)$$

<u>for bosons</u>, while <u>for fermions</u> we obtain in the same way

$$S_{\Delta w}(K_1,K_2,K',K'')=(\alpha/f)(A_1+A_2)<w(K_1,K')><w(K_2,K'')>\delta_{K',K''}(1/N-1). \qquad (53)$$

The last factors have been added in an attempt to generalize the results, derived here for N=2, to the case of N particles in the final state, on the basis of the result of the preceding Sections II.2 and II.3.

II.7 EFFECT OF A FINITE MEAN FREE PATH ON QUANTUM 1/f NOISE

The very low frequency radiation emitted by current carriers in condensed matter is important for the calculation of quantum 1/f noise. Here the spontaneously emitted radiation is calculated for a carrier subject to a large number of closely consecutive scattering processes. The result of the calculation is similar to the radiaiton from a single representative scattering process, with a correction factor of the order of unity.

In this Section we show that conventional quantum 1/f noise from a large number of closely consecutive, random, scattering events can be calculated from the average properties of a single scattering process, and is close to the quantum 1/f noise of an isolated scattering cross section in the case of randomizing collisions.

Here we calculate the extreme low frequency (e.l.f.) radiation from a large number N of very frequent consecutive scattering events and compare with what we would expect from a single isolated scattering of

the current carrier. It is well known that these radiation energy losses also determine the conventional quantum 1/f noise[1-8].

We start with a review of the derivation of the power radiated in a collision with acceleration v of the carrier of charge e:

$$\mathbf{A} = (-4\pi/c)\mathbf{j}, \quad (54)$$

$$\mathbf{A}(x,t) = (1/c)\int\{\mathbf{J}[\mathbf{x}',t - (|\mathbf{x} - \mathbf{x}'|/c)]/|\mathbf{x} - \mathbf{x}'|\}d^3x'$$
$$= (1/cr)\int\mathbf{J}[(\mathbf{x}',t - |\mathbf{x}' - \mathbf{x}|/c)]d^3x'; \quad (55)$$

$$\mathbf{E} = -(1/c)\mathbf{A} = e\mathbf{v}/c^2r, \qquad \mathbf{B} = (\mathbf{k}/|\mathbf{k}|)\times\mathbf{E}; \quad (56)$$

$$\mathbf{Y} = (c/4\pi)\mathbf{E}\times\mathbf{H}, \qquad P = \int\mathbf{Y}d\mathbf{S} = (2e^2/3c^3)v^2. \quad (57)$$

For one collision we separate a totally random part of zero average $\Delta\mathbf{v}$, and an average part proportional with the previous velocity, in the total velocity change $(\Delta\mathbf{v})_{tot}$:

$$\mathbf{v} = \delta(t)(\Delta\mathbf{v})_{tot}$$
$$(\Delta\mathbf{v})_{tot} = \Delta\mathbf{v} - \gamma\mathbf{v} \quad \text{(Definition of } \gamma\text{);} \quad 0 < \gamma < 1. \quad (58)$$

We note that for randomizing collisions with little or no memory the randomness parameter γ approaches the unity from below, and is very close to unity. For N consecutive sudden accelerations we obtain for **v** a sum of N delta functions. Expanding the delta functions as Fourier integrals, the total energy emitted by one particle is

$$W = \int Pdt = (2e^2/3c^3\int dt|[(\Delta_1\mathbf{v})_{tot}\int e^{2\pi ift}df$$
$$+(\Delta_2\mathbf{v})_{tot}\int e^{2\pi if(t-t\ )}df + \ldots\ldots +(\Delta_N\mathbf{v})_{tot}\int e^{2\pi if(t-t\ \ )}\ ]|^2. \quad (59)$$

Neglecting the short times between collisions, i.e., setting $t_1 = t_2 = \ldots\ldots = t_{N-1} = 0$, we obtain in the low frequency limit, i.e., if we do not care about errors in the integrands and in the resulting power spectral density as long as they are restricted to the high frequency region,

$$W = (2e^2/3c^3)\int dt\int df\int df' e^{2\pi i(f-f')t}$$
$$|\Delta_1\mathbf{v} - \gamma\mathbf{v} + \Delta_2\mathbf{v} - \gamma[\Delta_1\mathbf{v} + (1-\gamma)\mathbf{v}] + \Delta_3\mathbf{v} - \gamma\{\Delta_2\mathbf{v} + (1-\gamma)[\Delta_1\mathbf{v} + (1-\gamma)\mathbf{v}]\}$$
$$+\Delta_4\mathbf{v} - \gamma\{\Delta_3\mathbf{v} + (1-\gamma)[\Delta_2\mathbf{v} + (1-\gamma)(\Delta_1\mathbf{v} + (1-\gamma)\mathbf{v})]\} + \ldots\ldots|^2 \text{ for } f\to 0. \quad (60)$$

Summing the geometric series repeatedly,

$$W = (2e^2/3c^3)\int df|-\gamma \mathbf{v} \sum_{n=0}^{N-1}(1-\gamma)^n + \Delta_1\mathbf{v}\ [1 - \gamma \sum_{n=0}^{N-2}(1-\gamma)^n]$$
$$+ \Delta_2\mathbf{v}[1 - \gamma \sum_{n=0}^{N-3}(1-\gamma)^n] + \ldots\ldots.|^2$$
$$= (2e^2/3c^3)\int df|\mathbf{v}[1 - (1-\gamma)^N] + \Delta_1\mathbf{v}(1-\gamma)^{N-1} + \Delta_2\mathbf{v}(1-\gamma)^{N-2} + \ldots + \Delta_N\mathbf{v}|^2. \quad (61)$$

Finally, we take the square and we average over the independent random $\Delta\mathbf{v}$ vectors. Summing up geometric series again, we obtain for N going to infinity in the limit

$$\langle W\rangle = (2e^2/3c3)\int df\{\langle \mathbf{v}^2\rangle[1-(1-\gamma)^N]^2 + \langle(\Delta\mathbf{v})^2\rangle[1 - (1-\gamma)^{2N}]/[1 - (1-\gamma)^2]\}$$
$$\to (4e^2/3c^3)\int df\{\langle \mathbf{v}^2\rangle + \langle(\Delta\mathbf{v})^2\rangle/\gamma(2-\gamma)\} \quad (62)$$

The e.l.f. emission of radiation from a large number of statistically similar, closely spaced consecutive, random collisions of a current carrier is thus close to the emission from just one average collision or scattering process, as I had implied before.

The corresponding quantum 1/f effect in the current carried just by this carrier will therefore be[1-9]

$$S_{\delta j}(f)/(j^2) = 2(4e^2/3c^3hf)\{\langle \mathbf{v}^2\rangle + \langle(\Delta\mathbf{v})^2\rangle/\gamma(2-\gamma)\}$$
$$= (4\alpha/3\pi f)\{\langle \mathbf{v}^2\rangle/c^2 + \langle(\Delta\mathbf{v})^2\rangle/c^2\gamma(2-\gamma)\}. \quad (63)$$

The two terms in curly brackets should be roughly equal in average, in thermal equilibrium conditions. Indeed, by equating the average speed of the particle before and after a collision $\langle(\mathbf{v} - \gamma\mathbf{v} + \Delta\mathbf{v})^2\rangle = \langle \mathbf{v}^2\rangle$, we obtain $\langle \mathbf{v}^2\rangle = \langle(\Delta\mathbf{v})^2\rangle/[\gamma(2-\gamma)]$.

The randomness parameter γ can be replaced by a memory parameter $m = 1 - \gamma$, where again $0 < m < 1$. Then the corrective factor becomes $1/[\gamma(2-\gamma)] = 1/[1 - m^2]$. This shows that the corrective factor will not be less than unity.

For condensed matter applications, based on the classical concept of many consecutive random collisions of the same particle, one might question the low frequency content of the delta functions used above. In a quantum discussion, however, we can not identify the particles in general, and all we can say is that N scattering processes have taken place. Due to quantum exchange processes, the particles may have been an indefinitely long time in the initial state, and may persist in the final state an indefinite time. Even in the classical case, however, the low - frequency content of the delta functions used in this Section is limited only by the reciprocal observation time T^{-1}, because the velocity of the carrier is well defined over the whole duration T of the experiment.

Note that for randomizing collisions with little or no memory the corrective factor $1/[\gamma(2 - \gamma)]$ is close to unity. This proves that conventional quantum 1/f noise can in many cases be applied without including the finite mean free path correction.

II.8 CHARACTERISTIC FUNCTIONAL OF PHYSICAL THERMAL NOISE WHICH INCLUDES EQUILIBRIUM 1/F NOISE

In a published paper[6] the characteristic functional of quantum 1/f noise was derived. This result was applicable to quantum 1/f noise in any cross section or process rate, and in currents or voltages. In an early paper[26] the application of quantum 1/f noise to thermal equilibrium noise was performed, and the quantum 1/f Nyquist theorem was formulated. Deviations from the Gaussian form of thermal noise were expressed[27] in terms of the characteristic function.

The present Section reports the final step in the study of thermal noise with infrared radiative corrections, by presenting the characteristic functional of physical thermal noise. We call it "physical" because we refer to the actually observed thermal equilibrium (Johnson) noise, rather than to the strictly Gaussian noise expected from the Nyquist formula without infrared radiative corrections. The small quantum 1/f contributions come in as time-dependent infrared radiative corrections, required by the interaction of the current carriers with the electromagnetic field, or by the reaction of the bremsstrahlung back on

the current which has produced it. Knowing the characteristic functional of a process is the best we can do, the highest level and most comprehensive and complete description possible for a random process.

In terms of current fluctuations in equilibrium, to derive the characteristic functional of the physical thermal noise variable ξ, we express it in terms of the unmodulated theoretical Nyquist noise variable x:

$$\xi = x(G/G_o)^{1/2} = x + xy, \tag{64}$$

where $y = \delta G/G$ is the fractional fluctuation in the conductivity G of the conductor whose thermal equilibrium current fluctuations we are considering. Let the quantum 1/f noise variable y obey a Gaussian amplitude distribution of dispersion σ_2 and x one of dispersion σ_1. In terms of the zero-order Bessel function of imaginary argument, the product z=xy will have an amplitude distribution $P(z)=(1/\pi\sigma_1\sigma_2)K_o(z/\sigma_1\sigma_2)$ which has an elementary characteristic function

$$X(v) = (1 + \sigma_1^2\sigma_2^2v^2)^{-1/2}. \tag{65}$$

Although z and x are not independent, we have been able to calculate the characteristic function of ξ as an elementary function in the form

$$X(k) = (1 + k^2\sigma_1^2\sigma_2^2)^{-1/2} \exp[-k^2\sigma_1^2/2(1 + k^2\sigma_1^2\sigma_2^2)], \tag{66}$$

while the amplitude distribution itself can not be expressed in elementary functions.

In the second part of our work [#31, p.91] we derive the characteristic functional of quantum 1/f noise, and we familiarize the reader with characteristic functionals in general. Finally, using also Eq. (66), we derive the characteristic functional of physical thermal noise in the form

$$F[k(f)] = \exp\{-(1/2)\int \ln[1 + 8k_BTG(\alpha A/fN)(f/f_o)^{\alpha A}k^2(f)]df\}$$

$$\exp\{-2k_BTG\int k^2(f)[1 + 8k_BTG(\alpha A/fN)(f/f_o)^{\alpha A}k^2(f)]^{-1}df\}. \tag{67}$$

Here αA is the infrared exponent $\alpha A=(4\alpha/3\pi)<(\Delta v/c)^2>$, which is known to enter the expression of both the basic quantum 1/f noise formula and the characteristic functional of quantum 1/f noise in two ways: as a coefficient and as an additional exponent of the frequency f. Sommerfeld's fine structure constant $\alpha=e^2/hc=1/137$ is well known, and so is the Boltzmann constant k_B. Eq. (4) gives the characteristic functional of physical thermal noise for a sample containing N current carriers with an average velocity change $<(\Delta v)^2>$ in the scattering processes which determine their mobility.

II.9 COHERENT STATES AND CONVENTIONAL QUANTUM 1/f EFFECT

An electrically charged particle includes the bare particle and its field. The field has been shown in the last two decades to be in a coherent state, which is not an eigenstate of the Hamiltonian. Consequently, the physical particle is not described by an energy eigenstate, and is therefore not in a stationary state. In this Section we show that the *fluctuations arising from this non - stationarity* have a 1/f spectral density and affect the ordered, collective, or translational motion of the current carriers. This "coherent" quantum 1/f noise should be present along with the familiar quantum 1/f effect of elementary cross sections and process rates introduced ten years ago, just as the magnetic energy of a biased semiconductor sample coexists with the kinetic energy of the individual, randomly moving, current carriers. The amplitude of the quantum 1/f effect is always the difference of the coherent quantum 1/f noise amplitudes in the "out" and "in" states of the process under consideration and dominates in small samples, while large samples should exhibit the larger coherent quantum 1/f noise.

A physical, electrically charged, particle should be described in terms of coherent states of the electromagnetic field, rather than in terms of an eigenstate of the Hamiltonian. This is the conclusion obtained from calculations[28] of the infrared radiative corrections to any process performed both in Fock space (where the energy eigenstates are taken as the basis, and the particle is considered to have a well defined energy) and

in the basis of coherent states. Indeed, all infrared divergences drop out already in the calculation of the matrix element of the process considered, as it should be according to the postulates of quantum mechanics, whereas in the Fock space calculation they drop out only a posteriori, in the calculation of the corresponding cross section, or process rate. From a more fundamental mathematical point of view, both the description of charged particles in terms of coherent states of the field, and the undetermined energy, are the consequence of the infinite range of the Coulomb potential[29]. Both the amplitude and the phase of the physical particle's electromagnetic field are well defined, but the energy, i.e. the number of photons associated with this field, is not well defined. The indefinite energy is required by Heisenberg's uncertainty relations, because the coherent states are eigenstates of the annihilation operators, and these do not commute with the Hamiltonian.

A state which is not an eigenstate of the Hamiltonian is nonstationary. This means that we should expect fluctuations in addition to the (Poissonian) shot noise to be present. What kind of fluctuations are these? This question was answered in a published paper[11]. The additional fluctuations were identified there as 1/f noise with a spectral density of $2\alpha/\pi f$ arising from each electron independently, where $\alpha = 1/137$ is the fine structure constant. We will briefly derive this result again in the present Section, but we will stress the connection between the coherent quantum 1/f noise and the usual quantum 1/f effect.

Coherent Quantum 1/f Noise

The coherent quantum 1/f noise will be derived again in three steps: first we consider just a single mode of the electromagnetic field in a coherent state and calculate the autocorrelation function of the fluctuations which arise from its nonstationarity. Then we calculate the amplitude with which this mode is represented in the field of an electron. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the electromagnetic field be characterized by the wave vector q, the angular frequency $\omega = cq$ and the polarization λ.

Denoting the variables q and λ simply by q in the labels of the states, we write the coherent state[28,29,11] of amplitude $|z_q|$ and phase arg z_q in the form

$$|z_q> = \exp[-(1/2)|z_q|^2] \exp[z_q a_q^+] |0>$$

$$= \exp[-(1/2)|z_q|^2] \sum_{n=0}^{\infty} (z_q{}^n)/n! |n>. \qquad (68)$$

Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n> = (2^n n! \sqrt{\pi})^{-1/2} \exp[-x^2/2] H_n(x) e^{in\omega t}. \qquad (69)$$

This yields for the coherent state $|z_q>$ the representation

$$\Psi_q(x) = \exp[-(1/2)|z_q|^2]\exp[-x^2/2] \sum_{n=0}^{\infty} \{[z_q e^{i\omega t}]^n/[n!(2^n\sqrt{\omega})]^{1/2}\} H_n(x)$$

$$= \exp[-(1/2)|z_q|^2]\exp[-x^2/2]\exp[-z^2 e^{-2i\omega t} + 2xze^{i\omega t}]. \qquad (70)$$

In the last form the generating function of the Hermite polynomials was used[11]. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time t or the phase of z_q, is, for $|z_q|<<1$,

$$P_q(\tau,x) = <|\Psi_q|_t{}^2 \mid \Psi_q|_{t+\tau}{}^2 >$$

$$= \{1 + 8x^2|z_q|^2[1 + \cos \omega\tau] - 2|z_q|^2\}\exp[-x^2/2]. \qquad (71)$$

Integrating over x from $-\infty$ to ∞ , we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2 \cos \omega\tau\} \qquad (72)$$

This result shows that the probability contains a constant background with small superposed oscillations of frequency ω. Physically, the small

oscillations in the total probability describe a particle which has been emitted, or created, with a slightly oscillating rate, and which is more likely to be found in a measurement at a certain time than at other times in the same place. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations.

We now determine the amplitude z_q with which the field mode q is represented in the physical electron. One way to do this[11] is to let a bare particle dress itself through its interaction with the electromagnetic field, i.e. by performing first order perturbation theory with the interaction Hamiltonian

$$H' = A_\mu j^\mu = -(e/c)\mathbf{v}\cdot\mathbf{A} + e\phi \, , \tag{73}$$

where $\mathbf{A}$ is the vector potential and ϕ the scalar electric potential. Another way is to Fourier expand the electric potential $e/4\pi r$ of a charged particle in a box of volume V. In both ways we obtain[11]

$$|z_q|^2 = (e/q)^2(\hbar cqV)^{-1}. \tag{74}$$

Considering now all modes of the electromagnetic field, we obtain from the single - mode result of Eq. (72)

$$A(\tau) = C\,\Pi_q\{1 + 2|z_q|^2\cos\omega_q\tau\} = C\{1 + \Sigma_q\, 2|z_q|^2\cos\omega_q\tau\}$$

$$= C\{1 + 2(V/2^3\pi^3)\int d^3q\; |z_q|^2\cos\omega_q\tau\} \tag{75}$$

Here we have again used the smallness of z_q and we have intoduced a constant C. Using Eq. (74) we obtain

$$A(\tau) = C\{1 + 2(V/2^3\pi^3)(4\pi/V)(e^2/2hc)\int(dq/q)\cos\omega_q\tau\}$$

$$= C\{1 + 2(\alpha/\pi)\int\cos(\omega\tau)d\omega/\omega. \tag{76}$$

Here $\alpha = e^2/4\pi hc$ is the fine structure constant 1/137. The first term in curly brackets is unity and represents the constant background, or the d.c. part. The autocorrelation function for the relative, or fractional density fluctuations, or for current density fluctuations in the beam of charged particles is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khintchine theorem, the coefficient of cos is the spectral density of the fluctuations, $S_{|\psi|^2}$, or S_j for the current density $j=e(k/m)|\psi|^2$

$$S_{|\psi|^2}\langle|\psi|^{-2}\rangle = S_j\langle j\rangle^{-2} = 2(\alpha/\pi fN) = 4.6\cdot 10^{-3}\, f^{-1}N^{-1} \qquad (77)$$

Here we have included the total number N of charged particles which are observed simultaneously in the denominator, because the noise contributions from each particle are independent. This result is related to the well known conventional Quantum 1/f Effect[1-2]. If a beam of charged particles is scattered, passes from one medium into another medium (e.g. at contacts), is emitted, or is involved in any kind of transitions, the amplitudes z_q which describe its field will change. Then, even if the initial state was prepared to have a well-determined energy, the final state will have an indefinite energy, with an uncertainty determined by the difference between the new and old z_q amplitudes Δz_q. This, however, is just the bremsstrahlung amplitude Δz_q. We thus regain the familiar quantum 1/f effect, according to which the small energy losses from bremsstahlung of infraquanta yield a final state of indefinite energy, and therefore lead to fluctuations of the process rate, or cross section, of the process in which the electrons have participated, and which has occasioned the bremsstrahlung in the first place. The calculation of piezoelectric 1/f noise[30] which deals with phonons as infraquanta, was phrased in terms of the coherent field amplitudes z_q for the first time, although it is concerned only with the usual quantum 1/f effect. It has α substituted by the piezoelectric coupling constant g.

Connection with the Usual Quantum 1/f Effect

The assumptions included in the derivation of the above coherent quantum 1/f noise result are :

1 - The "bare particle" does not have compensating energy fluctuations which could cancel the fluctuations present in the field. The latter are due to the interaction with distant charges, and have nothing to do with the bare particle. Therefore, this assumption is quite reasonable.

2 - The experimental conditions do not alter the physical definition of the charged particle as a bare particle dressed by a coherent state field. This second assumption depends on the experimental conditions.

One way to understand this second assumption is based on the spatial extent of the beam of particles or of the physical sample containing charged particles, and is specifically based on the number of particles per unit length of the sample. According to this model, the coherent state in a conductor or semiconductor sample is the result of the experimental efforts directed towards establishing a steady and constant current, and is therefore the state defined by the collective motion, i.e. by the drift of the current carriers. It is expressed in the Hamiltonian by the magnetic energy E_m, per unit length, of the current carried by the sample. In very small samples or electronic devices, this magnetic energy

$$E_m = \int(B^2/8\pi)d^3x = [nevS/c]^2 \ln(R/r) \qquad (78)$$

is much smaller than the total kinetic energy E_k of the drift motion of the individual carriers

$$E_k = mv^2/2 = nSmv_2/2 = E_m/s. \qquad (79)$$

Here we have introduced the magnetic field B, the carrier concentration n, the cross sectional area S and radius r of the sample, the radius R of the electric circuit, and the "coherent ratio"

$$s = E_m/k = 2ne^2S/mc^2 \ln\,(R/r) \approx 2e^2N'/mc^2, \qquad (80)$$

where $N' = nS$ is the number of carriers per unit length of the sample and the natural logarithm $\ln(R/r)$ has been approximated by one in the last form. We expect the observed spectral density of the mobility fluctuations to be given by a relation of the form

$$(1/\mu^2)S_\mu(f) = [1/(1+s)][2\alpha A/fN] + [s/(1+s)][2\alpha/\pi fN] \qquad (81)$$

which can be interpreted as an expression of the effective Hooge constant if the number N of carriers in the (homogeneous) sample is brought to the numerator of the left hand side. Eq. (81) needs to be tested experimentally. In this equation $\alpha A = 2\alpha(\Delta v/c)^2/3\pi$ is the usual nonrelativistic expression of the infrared exponent, present in the familiar form of the quantum 1/f effect [4-9]. This equation does not include the quantum 1/f noise in the surface and bulk recombination cross sections, in the surface and bulk trapping centers, in tunneling and injection processes, in emission or in transitions between two solids.

Note that the coherence ratio s introduced here equals the unity for the critical value $N' = N'' = 2\cdot 10^{12}$/cm., e.g. for a cross section $S = 2\cdot 10^{-4}$ cm^2 of the sample when $n = 10^{16}$. For small samples with $N' \ll N''$ only the first term survives, and for $N' \gg N''$ only the second term remains in Eq. (81). We hope that an expression similar to Eq. (81) will allow us to extend the present good agreement between theory and experiment to the semiconductor samples of intermediate size.

II.10 COHERENT STATE PIEZOELECTRIC QUANTUM 1/f NOISE

Coherent state quantum 1/f noise has become known[11,12] as the fluctuation caused by the nonstationary character of electrically charged particles or current carriers in solids. This nonstationarity appears because the coherent state of any harmonic oscillator, in particular also the state of a physical charged particle whose electromanetic field is known to be in a coherent state, is not an eigenstate of the Hamiltonian, but rather an eigenstate of the field operator, with well defined amplitude, but with an uncertain photon number (see Sec. II.9).

On the other hand, the similarity of the electron - photon interaction to the piezoelectric interaction between electrons and transversal photons has led us[30] to apply the conventional quantum 1/f theory to the case of piezoelectric coupling. The result of that calculation is a rather large quantum 1/f noise which we call piezoelectric quantum 1/f noise, in qualitative agreement with the very large values given by the measurements[31,32].

It is therefore worthwhile to investigate the applicability of the concept of coherent states 1/f noise to the case of transversal phonons as infraquanta. In this Section we calculate for the first time the coherent state quantum 1/f noise caused by phonons as infraquanta.

Starting from the expression[30]

$$A_{\mathbf{q}}(\mathbf{k}) = (2\pi g/qV)^{1/2}\,(v_s/q)\,[\hbar q/2m - \hbar \mathbf{q}_o\cdot\mathbf{k}/m + v_s]^{-1} \qquad (82)$$

of the amplitude of the phonon state q associated with the coherent state which describes the deformation field of an electron of wave vector **k,** we obtain the number of phonons emitted when the state was created, by squaring this amplitude and summing over the phonon modes:

$$\begin{aligned} N_{ph}{}^{coh} &= (2)[V/(2\pi)^3]\int d^3q\,(2\pi g v_s{}^2/Vq^3)[v_s - \hbar \mathbf{q}_o\cdot\mathbf{k}/m]^{-2} \\ &= (2)\,(g/\pi)\,[1-(\hbar k/mv_s)^2]^{-1}\int dq/q. \end{aligned} \qquad (83)$$

This is also the number of phonons obtained by expanding the lattice strain (or deformation) field of the electron in terms of phonon modes. Here g is the piezoelectric coupling constant, V the volume of a normalization box which disappears later in the calculation, v_s the speed of sound, $\mathbf{q}_o = \mathbf{q}/q$ a unit vector, and the factor 2 placed in brackets accounts for the two polarization modes associated with each phonon vector **q**. The angular integration was performed with the assumption $\hbar k/mv_s < 1$, i.e., for subsonic electrons. For higher momenta no 1/q spectrum is obtained, and so no 1/f contributions are added. This implies a decrease of the 1/f noise at supersonic drift velocities of the carriers, compared to the extrapolation from the subsonic region.

According to the general quantum 1/f theoretical relation[2], the phonon number spectral density given by the last equation is also half the spectral density of fractional coherent state quantum 1/f current fluctuations observable in a beam of electrons of wave vector **k**:

$$S_j(f)/j^2 = 4\ (g/f)\ [1-(\hbar k/mv_s)^2]^{-1}. \tag{84}$$

This expression differs from our previous (piezoelectric conventional quantum 1/f) result by the absence of the factor $(1/3)[\hbar(\mathbf{k} - \mathbf{k}')/mv_s]^2$.

A more detailed presentation of this derivation is found in a recently published article[33]

III. APPLICATIONS OF THE QUANTUM 1/F THEORY

III.1 DERIVATION OF MOBILITY QUANTUM 1/f NOISE IN n^+-p DIODES

For a diffusion limited n^+-p junction the current is controlled by diffusion of electrons into the p - region over a distance of the order of the diffusion length $L = (D_n\tau_n)^{1/2}$ which is shorter than the length w_p of the p - region in the case of a long diode. If N(x) is the number of electrons per unit length and D_n their diffusion constant, the electron current at x is

$$I_{nd} = -\ eD_n dN/dx, \tag{85}$$

where we have assumed a planar junction and taken the origin x = 0 in the junction plane. Diffusion constant fluctuations, given by kT/e times the mobility fluctuations, will lead to local current fluctuations in the interval Δx

$$\delta\Delta I_{nd}(x,t) = I_{nd}\Delta x \delta D_n(x,t)/D_n. \tag{86}$$

The normalized weight with which these local fluctuations representative of the interval Δx contribute to the total current I_d through the diode at x

$= 0$ is determined by the appropriate Green function and can be shown to be $(1/L)\exp(-x/L)$ for $w_p/L >> 1$. Therefore the contribution of the section Δx is

$$\delta \Delta I_d(x,t) = (\Delta x/L)\exp(-x/L) I_{nd} \delta D_n(x,t)/D_n, \tag{87}$$

with the spectral density

$$S_{\Delta I_d}(x,f) = (\Delta x/L)^2 \exp(-2x/L)\ I_{nd}^2\ S_{D_n}(x,f)/D_n^2. \tag{88}$$

For mobility and diffusion fluctuations the fractional spectral density is given by $\alpha_{Hnd}/fN\Delta x$, where α_{Hnd} is determined from quantum 1/f theory according to Sec III.3. With Eq. (85) we obtain then

$$S_{\Delta I_d}(x,f) = (\Delta x/L^2)\ \exp(-2x/L)\ (eD_n dN/dx)^2\ \alpha_{Hnd}/fN. \tag{89}$$

The electrons are distributed according to the solution of the diffusion equation, i.e.

$$N(x) = [N(0) - N_p]\ \exp(-x/L); \quad dN/dx = -\{[N(0) - N_p]/L\}\ \exp(-x/L). \tag{90}$$

Substituting into Eq. (89) and simply summing over the uncorrelated contributions of all intervals Δx we obtain

$$S_{I_d}(f) = \alpha_{Hnd}(eD_n/L^2)^2 \int_0^{w_p} [N(0) - N_p]^2 e^{-4x/L} dx\ /\{[N(0) - N_p]\ e^{-x/L} + N_p\}. \tag{91}$$

We note that $eD_n/L^2 = e/\tau_n$. With the expression of the saturation current $I_0 = e(D_n/\tau_n)^{1/2}N_p$ and of the current $I = I_0[\exp(eV/kT) - 1]$, we can carry out the integration

$$S_{I_d}(f) = \alpha_{Hnd}(eI/f\tau_n) \int_0^1 a^2u^3du/(au + 1) = \alpha_{Hnd}(eI/f\tau_n)\ F(a). \tag{92}$$

Here we have introduced the notations

$u = \exp(-x/L), \quad a = \exp(eV/kT) - 1,$

$$F(a) = 1/3 - 1/2a + 1/a^2 - (1/a^3)\ln(1+a). \tag{93}$$

Eq. (92), obtained by van der Ziel and Anderson, gives the diffusion noise as a function of the quantum 1/f noise parameter α_{Hnd}. A similar result can be derived for the quantum 1/f fluctuations of the recombination rate in the bulk of the p - region.

III.2. OVERVIEW OF QUANTUM 1/f NOISE IN n+p JUNCTIONS AND MIS DIODES

As we have seen in the Sec. II, quantum 1/f noise is a low-frequency fluctuation process present in elementary cross sections σ and process rates Γ, given by the fractional spectral density

$$S_\sigma(f)/\sigma^2 = S_\Gamma(f)/\Gamma^2 = (4\alpha/3\pi fN)(\Delta \mathbf{v}/c)^2 \tag{94}$$

for conventional quantum 1/f noise which is applicable to small devices, and

$$S_\sigma(f)/\sigma^2 = S_\Gamma(f)/\Gamma^2 = (2\alpha/\pi fN) = \alpha_c/fN \tag{95}$$

for coherent state quantum 1/f noise which is applicable to large devices in which the energy of the carrier drift motion is predominantly magnetic, rather than kinetic. Here $\Delta \mathbf{v}$ is the velocity change of the carriers in the processes considered, $\alpha = 1/137$ is the fine structure constant, N the number of carriers simultaneously interrogating the cross sections or process rates, $\alpha_c = 4.6\ 10^{-3}$ the coherent quantum 1/f noise coefficient, and c the speed of light. The two forms of quantum 1/f noise are closely related infrared divergence phenomena which arise due to the interaction of electrons and soft photons, as we have seen in Sec. II.9.

Both in n^+p diodes[13,16-18,23,34] and metal-insulator-semiconductor[35,36] (MIS) devices the current will be determined by cross sections σ_i such as recombination and scattering cross sections (by phonons and

lattice defects), as well as by other process rates (e.g., band to band and trap-assisted tunneling, particularly important in long-wavelength $Hg_{1-x}Cd_xTe$ detectors). The spectral density of the resulting quantum 1/f current fluctuations is therefore

$$S_I(f) = \Sigma_i [\partial I/\partial \sigma_i]^2 S_{\sigma i}(f) + \Sigma [\partial I/\partial \Gamma_i]^2 S_{\Gamma i}(f), \qquad (96)$$

where the spectral densities on the right hand side will be given by Eqs. (94) or (95), depending on the size of the device.

A. N+P Junctions

The current density I of Eq. (96) contains a diffusion term I_d, a term I_r caused by recombination in the space charge region, a surface recombination term I_s, a tunneling term I_t and a photovoltaic term caused by the creation of electron hole pairs by photons:

$$I = I_d + I_r + I_s + I_t + q\eta\Phi$$

$$= qn_i\{(n_i/n_0)(D_n/\tau_n)^{1/2}(e^{qV/kT}-1) + (W/\tau)(e^{qV/2kT}-1) + s\} + I_t + q\eta\Phi. \qquad (97)$$

Here n_i is the intrinsic concentration, n_0 the concentration of acceptors on the p side, D_n and τ_n the diffusion constant and lifetime of minority carriers on the p side, W the width of the depletion region, $\tau = \tau_{po} + {}_{no}$ the Shockley-Hall-Read lifetime, V the applied voltage, s the surface recombination speed, η the quantum efficiency and Φ the incident flux of photons. With the exception of the last term, the terms in Eq. (97) are known as dark current components.

The first term in rectangular brackets in Eq. (97) gives the diffusion current density I_d, and yields a noise term[16-18]

$$S_{Id}(f) = (\alpha_d+\alpha_r)(qI_d/f\tau_n)F(a); \quad \alpha_d = (4\alpha/3\pi)(h/m^*bc)^2\exp(-\theta/2T) \qquad (98)$$

in Eq. (96) for small devices, and

$$S_{I_d}(f) = \alpha_c(qI_d/f\tau_n)F(a) \qquad (99)$$

for large devices, with $F(a) = -1/3 -1/2a +1/a^2 -(1/a^3)\ln(1+a)$ and $a = \exp(qV/kT) -1$. Here b is the lattice constant, m^* the effective mass of the electrons, and θ the Debye temperature. These results are obtained by adding the scattering and recombination quantum 1/f cross section fluctuation spectra for all points on the p side of a long n^+p diode with the appropriate Green function weight $[\partial I/\partial\sigma_i]^2$, as prescribed by Eq. (96). The spatial dependence $\exp(-x/L_d)$ of this Green function is given by the diffusion equation applied to electrons on the p side, with L_d being the electron diffusion length. If the length w_p of the p side is shorter than L_d, we have to replace F(a) by $F(a) - F[a \exp(-w_p/L_d)]$ in Eqs. (98) - (99), and also to include the quantum 1/f noise of the recombination cross sections at the p contact.

Similarly we obtain[16,20]

$$S_{I_r} = \alpha_r qI_r \tanh(qV/2kT)/f\tau;$$
$$= (4\alpha/3\pi c^2)[2q(V_{diff} -V) +3kT]\{(m_n^*)^{1/2} +(m_p^*)^{1/2}\}^{-2}. \qquad (100)$$

The corresponding expression for the surface recombination current is given by Eq. (100) with half of the surface potential jump U added to the diffusion potential V_{diff}. The tunneling current I_t and its noise is the same as in MIS devices,and will be discussed below. In the final form of Eq. (96) the fine structure constant α appears as a general factor.

2. MIS Devices

The MIS infrared detector devices[35] considered here are homogeneous $Hg_{1-x}Cd_xTe$ slabs of thickness of the order of the minority carrier diffusion length, on which insulated metallic field effect plates are applied with a potential that creates a surface inversion layer. The transition between the bulk and the inverted surface layer is a pn junction which is similar to a conventional pn junction caused by inhomogeneous doping, connected in series with a capacitor, but is free of the defects introduced in the doping process. The theory summarized for junctions in

the previous Section is again applicable for the quantum 1/f noise of the (dark) current flowing through the junction, but this time the longer life times in the denominator of Eqs. (98), (99) and (100) will lead to lower quantum 1/f noise. This is a considerable advantage of MIS devices, but on the other hand, they have to be operated in a pulsed mode in which the photoelectrically generated carriers separated by the field of the induced junction, and accumulated in the potential well of the surface inversion layer, have to be removed and registered periodically, by alternating the inverting voltage pulses at the plate with short "readout" potentials which generate temporary conditions close to flat energy bands at the surface.

In addition to the photoelectrically generated carriers, the readout signal also contains the carriers accumulated through the various dark current mechanisms mentioned above. Therefore, at low frequencies the 1/f noise contained in the dark current I will limit the performance of MIS detectors as infrared detectors.

The integration implied by the pulsed mode of operation acts as a low pass filter which does not influence the signal to noise ratio at frequencies well below the readout frequency. Furthermore, the pulsed mode of operation does not affect the quantum 1/f noise, because the quantum 1/f fluctuations of cross sections and process rates are continuously tested by carriers in thermal agitation, and are present independently of the method of sampling used by us.

In general the dark current I can again be written in the form

$$I = I_d + I_r + I_s + I_t + q\eta\Phi. \tag{101}$$

The five terms on the right hand side correspond to minority carrier diffusion from the bulk, generation from Shockley-Read centers in the depletion region, generation from SR centers at the surface, tunneling, and photoelectric generation by the thermal radiation background flux Φ. We shall give the formulae[20] derived in detail in Sec. III.4, which determine these terms below, also including an example of their calculation in a p-type device

$$I_d = (qn_i^2/p_o)[kT\mu_n/q\tau_n]^{1/2} = (1.6\ 10^{-19}C\ 36\ 10^{24}\ cm^{-6}/10^{15}cm^3)\ [0.01V\ 1.5\ 10^5(cm^2/Vs)/10^{-6}s]^{1/2} = 2\ 10^{-4}A/cm^2, \quad (102)$$

$$I_r = qn_iW/2\tau_0 = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ 2\ 10^{-4}cm\ )/(2\ 10^{-6}s)] = 10^{-4}A/cm^2 \quad (103)$$

$$I_s = J_b = qn_is/2 = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ 20cm/s] = 1.8\ 10^{-5}A/cm^2, \quad (104)$$

$$I_t=10^{-13}(N_r\Phi_s/E_g)\exp[-5.3\ 10^6E_g^2/E]=10^{-13}(4\ 10^{12}cm^{-3}0.2V/0.062V)$$

$$\exp[-5.3\ 10^6(0.062V)^2/3000V/cm] = 8\ 10^{-4}A/cm^2, \quad (105)$$

Here $n_i = 6\ 10^{12}cm^3$ is the intrinsic carrier concentration, and the concentration of holes was taken to be $p_o = 10^{15}cm^{-3}$. The mobility μ = $1.5\ 10^{15}cm^2/Vs$ as well as the life time $\tau_n = 10^{-6}s$ of the minority carriers have to be replaced by μ_p and τ_p for the case of n-type devices. The surface recombination speed was denoted by s = 40cm/s, and the concentration of intermediary states effective for tunneling was denoted by $N_r = 4\ 10^{12}cm^{-3}$. The band gap considered was $E_g = 0.065eV$, the surface potential $\Phi_s = 0.2V$ and the electric field below the surface E = 3000V/cm. All numerical values have been included only as an example and are not characteristic of a particular device. The numerical factors in Eq. (105) correspond to p - type HgCdTe with a 20 μm cutoff wavelength and were taken from the paper of Kinch and Beck[36]. For n-type devices we also need to include tunneling via surface states. If the density of fast surface states is denoted by $N_{fs} = 10^{12}\ cm^{-2}V^{-1}$, the current generated from tunneling via a uniform density of fast surface states across the band gap will be given by[35]

$$I_{tsc} = -qp_1N_{fs}kT/2q = 1.75\ 10^{-4}\ A/cm^2. \quad (106)$$

The sum of the first four currents on the right hand side of Eq. (101) must be smaller than the fifth which corresponds to the thermal radiation background flux, for background limited (BLIP) performance. Although not all terms in the dark current are of importance, we still retain them,

because their quantum 1/f noise may be quite significant, even if the corresponding current is negligible. We shall now proceed with the calculation of quantum 1/f noise contributions from all these currents.

The quantum 1/f noise in I_d and I_r is the same as in n^+p diodes, and is given by Eqs. (98)-(100).

The surface recombination current is a dark current component originating from surface states at the interface between the surface passivation layer and the bulk. The recombination cross sections of these states will exhibit quantum 1/f noise. The quantum 1/f noise coefficient α_H reflects the velocity change of carriers involved in this process

$$\alpha_H = (4\alpha/3\pi)(2/m^*c^2)[3kT/2 + eU/2 + 0.1eV], \qquad (107)$$

where U is the surface potential jump present between the surface passivation layer and the bulk even if no gate voltage is applied, and V is the applied gate voltage which we take with a coefficient less than unity, here for example with a weight of 0.1.

The calculation of surface recombination quantum 1/f noise is similar to the calculation of quantum 1/f noise from recombination in the space charge region. However, in this case the cross sections are not distributed over the width of the junction, but rather are concentrated at the surface which is characterized by the surface potential Φ_s. Therefore, with $x=eV/2kT$, we can write an expression of the form

$$S_{Is}(f) = \alpha_H e I_s \tanh x \,/f(\tau_{no} + \tau_{po})\exp(e\Phi_s/kT). \qquad (108)$$

We note that the fabrication process of MIS structures introduces less bulk defects than the fabrication of photovoltaic devices. Nevertheless, the fabrication of MIS devices introduces some defects in the bulk layer located right under the surface. These defects will manifest themselves through a contribution to indirect tunneling.

Quantum 1/f Noise in the Tunneling Rate. The tunneling component I_t of the dark current is particularly important in narrow band gap material. In the case of tunneling from a surface accumulation layer to the bulk, the

velocity change of the carriers will lead us from the thermal velocity of the carrier on one side of the barrier to the thermal velocity of a carrier on the other side of the barrier, if band to band tunneling is considered. If, however, the tunneling goes via intermediate states located in the band gap, the velocity is zero as long as the carrier is stationary in the intermediate state. We can therefore write the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)6kT/m^*c^2 \tag{109}$$

for band to band tunneling, and

$$\alpha_H = (4\alpha/3\pi)3kT/m^*c^2 \tag{110}$$

for tunneling via intermediate states in the band gap, where we have considered the average squared velocity change two times smaller. The effective mass is the mass of the minority carriers in the bulk material.

For band to band tunneling from a surface inversion layer to the bulk, the velocity change of carriers corresponds to an energy difference of the *order of the band gap* E_g *plus an energy difference of the order of the* thermal energy 3kT/2, provided we are dealing with deep inversion, as used in practical MIS devices in the pulsed mode of operation. This yields the quantum 1/f coefficient

$$\alpha_H = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 \tag{111}$$

for band to band tunneling. For tunneling via intermediate states in the band gap the corresponding energy difference will be smaller, and therefore we replace E_g by $E_g/2$

$$\alpha_H = (4\alpha/3\pi)(E_g/2 + 3kT/2)m^*c^2 \tag{112}$$

This relation is applicable both if the intermediate states are located in the depletion region or at the surface.

In the last four equations we did not divide by the number of carriers simultaneously involved in the tunneling process, because this number is

less than unity for practical tunneling currents. Whenever a cross section or a process rate is tested with one electron or less than one electron at a time, the effective number of electrons in the denominator of the quantum 1/f formula must be replaced by unity. Let us calculate the average number of carriers simultaneously present in the tunneling process at any time. The tunneling process occurs over a distance $d = E_g/eE$, and the speed v of the carriers will be of the order of the thermal speed in the case of an accumulation layer, and of the order of the band gap energy in the case of a surface inversion layer. Dividing d by v, we obtain a tunneling time $t = 6\ 10^{-13}$s for accumulation layers and $t = 3\ 10^{-13}$s for inversion layers. From Eq. (105) we know that the tunneling current is of the order of 10^{-3}A/cm^2. Multiplying this by t, we obtain 3 - 6 10^{-16} C, i.e., 2000 - 3000 electrons/cm^2 tunneling simultaneously in a device of 1 cm^2 area. In a device of dimensions 50 μm x 50 μm = 2.5 10^{-5} cm^2 the average number of carriers in the process of tunneling at any time is therefore 0.05 - 0.075, and this is indeed much less than unity. Nevertheless, if the area of the device exceeds 5 10^{-4} cm^2, Eqs. (109) - (112) require an additional factor e/tI_tA which makes the noise spectral density proportional to I_t and A, rather than to the square of these quantities.

The photoelectric current will reflect the fluctuations in the number of photons arriving from the radiation background. The quantum efficiency will not exhibit considerable quantum 1/f noise, because the generated carriers will be collected with certainty. Therefore the collection of photoelectrically generated carriers is not controlled by any cross section or process rate affected by considerable quantum 1/f noise. If we neglect the 1/f noise generated in the series resistance of the diode, there should be no photoelectrically generated 1/f noise from a short-circuited diode.

The resulting 1/f noise. Since the various dark current fluctuations with 1/f spectrum are statistically independent, the total 1/f noise is simply obtained by summing all contributions.

Let us now evaluate the spectral density S'(f) of fractional fluctuations in the various dark current noise contributions per cm^2 of transversal detector (or gate) area, including also a numerical example for a MIS infrared detector. For a given detector, this needs to be divided by

the area A of the detector to yield the corresponding fractional spectral densities: S(f) =[S'(f)]/A. Fractional fluctuations are dimensionless, so S'(f) will have the dimension of a reciprocal frequency, times a squared length unit which simplifies when we divide by the area of the detector at hand. Let S'_{Id} be the spectral density of fractional fluctuations in the noise caused by quantum 1/f fluctuations in diffusion, S'_{Ir} in bulk recombination, S'_{Is} in surface recombination, and S'_{It} in tunneling. With $m_p^* = 0.55\ m_o$, $m_n^* = 0.02\ m_o$, $\tau_n = 10^{-6}$s, $E_g = 0.1$ eV, 3kT/2 = 0.01 eV, we obtain for a p-type MIS device with $w_p \gg L_d$

$$S'_{Id} = (\alpha_{Hnd} + \alpha_{Hnr})[e/f\tau_n I_d]F(a) = \alpha_{coh}\{e^{1/2}/[f(kT\mu\tau_n)^{1/2}N_p]\}F(a)/a$$

$$=(4.6\ \ 10^{-3}/4fN_p)\ 4\ \ 10^{-10}C^{1/2}/[10^{-6}s\ \ 1.5\ \ 10^5(cm^2/Vs)\ \ 4\ \ 10^{-21}\ J]^{1/2}$$

$$=1.8\ \ 10^{-6}cm^2/f,\ [\text{or } 10^{-8}\ cm^2/f, \text{ with } \alpha_p = 2\ \ 10^{-5} \text{ for incoher. noise}] \qquad (113)$$

$$S'_{Ir} = [\alpha_{He}e/f(\tau_{no} + \tau_{po})I_r]\tanh x = [\alpha_{He}e/(feAwn\ \ \tanh x)]\tanh x$$
$$= \alpha_{He}/fAwn_i = 4.6\ \ 10^{-9}cm^2/f \qquad (x = eV/2kT) \qquad (114)$$

$$S'_{Is} = (4\alpha/3\pi)(2/m^*c^2)[3kT/2 + eU/2 + 0.1Ve]\ [e\ \tanh x\ /f(\tau_{no}+\tau_{po})I_s]$$

$$=(4\alpha/3\pi 0.02)(2/500{,}000)[0.025+0.5+0.5]\ \ [e\ \ \tanh x/feAwn_i(e^x-1)]$$
$$= 7\ \ 10^{-8}cm^2/f \qquad (115)$$

$$S'_{It} = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 = (4/9.5\ \ 137\ \ 0.02)(0.11/500{,}000)$$
$$= 3.3\ \ 10^{-8}cm^2/f \qquad (116)$$

S'_{Id} was calculated in the small bias limit for $w_p \gg L$, but w_p= 0.25 L gives the same result; the incoherent case with a lattice constant of b = 0.65 nm and θ = 320 K was also listed above, and would give 1.8 $10^{-10}cm^2/f$ for a n-type device. Eqs. (113) - (116) would be reduced m_p^*/m_n^* = 27.5 times for n-type devices. S'_{Is} has been calculated with the inclusion of a term of 10% of the applied gate voltage V (= 5 volt in our example) into the kinetic energy of the carriers at the surface.

We conclude that the effective mass present in the denominator leads to lower conventional quantum 1/f noise in n type devices.

III.3 QUANTUM1/F NOISE IN JUNCTION INFRARED DETECTORS

A. QUANTUM 1/f NOISE SOURCES APPLICABLE TO n^+p DIODES

The various quantum 1/f noise sources present in n^+p diodes which we will consider here, have in common that they can be described by Eq. (3) with a Δv^2 value described by an effective energy $E = qV_o$, where V_o is an effective potential which accelerates the carriers

$$(\Delta v^2)/c^2 = 2E/m^*c^2 = 2qV_o/m^*c^2. \qquad (117)$$

In various quantum 1/f noise sources the applicable velocity change v determines the Hooge constant:

(a) *Recombination quantum 1/f noise in the bulk space charge region*[16]:

$$S_I(f) = \alpha_H q|I_{gr}|[\tanh(qV/2kT)]/f(\tau_{no} + \tau_{po}); \qquad (118)$$

$$\alpha_H = (4\alpha/3\pi)[2q(V_{dif}-V) + 3kT]\{(m_n^*)^{1/2} + (m_p^*)^{1/2}\}^{-2}c^{-2},$$

where $I_{gr} = qAWn_i(e^{qV/2kT} - 1)/(\tau_{po} + \tau_{po})$ is the recombination current, τ_{no} and τ_{po} the Shockley-Hall-Read lifetimes, V the applied voltage and V_{dif} the diffusion potential of the junction. Introducing an "effective carrier number" $N_{eff} = |I_{gr}|(\tau_{no}+\tau_{po})/q[\tanh(qV/2kT)]$, Eq. (118) may be written in the form

$$S_I(f) = \alpha_H I_{gr}^2/fN_{eff}. \qquad (119)$$

(b) *Quantum 1/f noise in the surface recombination current of n^+-p diodes.* This effect is caused by quantum 1/f fluctuations of the surface recombination cross sections. The calculation is similar to the previous (bulk) case, but the GR process is localized at the surface, and the

additional electric field arising from the potential jump 2U at the interface between the bulk and the oxide and passivation layers will lead to increased velocity changes of the carriers in the recombination process and to larger $_H$ values. Including also the quantum 1/f mobility fluctuation noise of the spreading resistance caused by the concentration of generation and recombination currents at the intersection of the depletion region with the surface of the diode, we obtain a current noise contribution

$$S_I(f) = \alpha'_H q|I'_{gr}|[\tanh(qV/2kT)]/f(\tau'_{no} + \tau'_{po})$$

$$+ \alpha_\mu (I'_{gr})^2 /\pi nfP \ [\ln(A/W^2)]^2, \qquad (120)$$

$$\alpha'_H = (4\alpha/3\pi)[2q(V_{dif}+U-V)+3kT]\{(m_n^*)^{1/2} + (m_p^*)^{1/2}\}^{-2}c^{-2} \qquad (121)$$

Here the primed quantities refer to the surface, i.e. we have introduced the surface GR current I'_{gr}, the lifetimes in the vicinity of the surface and the α'_H parameter for surface recombination. P is the perimeter, A the area and W the width of the junction. The quantum 1/f mobility fluctuation part is expressed in terms of the global α_μ parameter which includes all types of scattering weighed with the appropriate mobility ratio factors. Introducing again an "effective carrier number" $N'_{eff} = |I'_{gr}|(\tau'_{no}+\tau'_{po})/q[\tanh(qV/2kT)]$, the first term of Eq. (120) may also be written in the form:

$$S_I(f) = \alpha'_H (I'_{gr})^2/fN'_{eff}. \qquad (122)$$

Due both to the surface potential jump 2U and the 1/f noise of the spreading resistance, the surface recombination current will be noisier than an equal bulk recombination current, and this is in agreement with the experimental data[34].

(c) *Injection - extraction quantum 1/f noise*[16], due to injection or extraction of carriers across barriers. In this case, for not too small currents

$$S_I(f) = \alpha_{Hni}|I|q/f\tau;$$

$$\alpha_{Hni} = (4\alpha/3\pi)[2q(V_{dif}-V)+3kT]/m_n^*c^2, \quad (123)$$

where I is the injected current, and is the time of passage of a carrier through the barrier region. Introducing again an effective carrier number $N_{eff} = I\tau/q$, Eq. (123) may be written in a more general form, valid also for very small I

$$S_I(f) = \alpha_{Hni} I^2/fN_{eff}. \quad (124)$$

Note that in each case N_{eff}, as long as it is larger than 1, is proportional to I (otherwise $N_{eff} = 1$, see below), and that α_H depends on bias.

(d) *Recombination in the p region of a long n^+-p diode* of length W >> $(D_n\tau_n)^{1/2}$ yields, as shown in Sec. III.1,

$$S_I(f) = \alpha_{Hnr}|I|qF(a)/f\tau_n = \alpha_{Hnr}I^2/fN_{eff}; \quad (125)$$

$$N_{eff} = |I|\tau_n/qF(a), \qquad \alpha_{Hnr} = (4\alpha/3\pi)[3kT/m_n^*c^2], \quad (126)$$

Here F(a) is given by Eq. (93):

$$F(a) = 1/3 - 1/2a + 1/a^2 - (1/a^3)\ln(1+a), \quad (127)$$

where

$$a = \exp(eV/kT) - 1. \quad (128)$$

In this case α_H is very small. Therefore this contribution will often be masked by mobility fluctuation 1/f noise.

(e) *Recombination quantum 1/f noise at the p - region contact.* The carriers lose an average energy 3kT/2 in the conduction band in the recombination process, gain an energy of the order of qE_g due to energy differences of the order of the band gap E_g accelerating the carriers at the contact surface, and gain an average energy 3kT/2 in the valence band, so that

$$\alpha_{Hnc} = (4\alpha/3\pi)[(2qE_g + 3kT)/m_n^*c^2]. \quad (129)$$

This effect will be important in short diodes ($W_p < L$), where the recombination speed at the contact partially determines the current through the device.

(f) *Quantum 1/f noise in the tunneling rate.* Tunneling is observed in n^+-p diodes with sufficient gate bias[34]. If we assume that the momentum change of the carriers in the tunneling process is of the order of the thermal r.m.s. momentum, we obtain a minimal quantum 1/f noise power spectrum

$$S_I(f) = \alpha_H I^2/fN_{eff}; \;\; N_{eff} = |I|\tau/q, \tag{130}$$

$$\alpha_H = (4\alpha/3\pi)[3kT/m_n{}^*c^2], \tag{131}$$

where is the time of passage through the barrier, or tunneling, i.e. the time during which each carrier contributes to the current through the barrier. Since the width of the barrier crossed by tunneling is small, this time is very short, of the order of 10^{-14}s. N_{eff} will then become larger than 1 at currents exceeding 10^{-5}A, leading to a linear current dependence of the noise power. At lower bias N_{eff} must be set equal to unity in Eq. (130). and this gives a quadratic current dependence. On the other hand, at large applied voltages, or for tunneling out of a metal, kT has to be replaced by a larger energy in the expression of α_H.

(g) *Mobility fluctuation 1/f noise in a long n+-p diode.* Although characterized mainly by a well defined momentum transfer of the carriers rather than by an effective energy, and although we have discussed it already above, this important process is listed here again. As was shown in Sec. III.1, this contribution can be written in the form

$$S_I(f) = \alpha_H |I| qF(a)/f\tau_n = \alpha_H I^2/fN_{eff}; \;\; N_{eff} = |I|\tau/qF(a), \tag{132}$$

where α_H is given by

$$\alpha_H = (4\alpha/3\pi)[h/m^*ac]^2 \exp(- \;/2T) + (4\alpha/3\pi)[6kT/mc^2] \tag{133}$$

for small devices, and is usually larger than in the case (d), particularly if the effective mass of the carriers is small. Here is the Debye temperature, a the lattice constant, and T the temperature. The first term in Eq. (133) is from Umklapp and inter - valley lattice scattering, while the second describes normal scattering processes. The function F(a) is again given by Eq. (93). In general this contribution includes Umklapp scattering, intervalley scattering, normal (non - Umklapp) phonon scattering, impurity scattering, and optical phonon scattering contributions in the quantum 1/f mobility fluctuation Hooge parameter . For large samples or devices, Eq. (95) is applicable. Eq. (81) is a suggestion for the transition region.

B DISCUSSION AND RECOMMENDATIONS

The quantum 1/f noise formulae presented above have been applied by Radford and Jones[34] to 1/f noise in GR, diffusion and tunneling currents in both double epitaxial layer and ion implanted n^+-p HgCdTe diodes. They obtained good agreement with the experimental data in general, but were a factor 20 below the measured values at positive gate bias, when an inversion layer formed at the surface. This discrepancy may be due both to the presence at positive gate bias of a noisy surface GR contribution (Eqs. 120-121), and to kinetic energies of the tunneling carriers above the thermal level in the vicinity of the inversion layer.

Another previously unexplained fact noted was the difference in the fractional noise level of surface and bulk recombination currents. This is caused in Eqs. (120-121) both by the surface potential jump 2U of the order of 1 Volt present at the interface between the bulk and the oxide and passivation layers, and by the quantum 1/f mobility fluctuation noise in the spreading resistance which affects the passage of carriers to and from the perimeter of the junction. Furthermore, the higher noise level of ZnS - passivated diodes may be caused by a larger surface recombination speed associated with these coatings compared to SiO_2 passivations, and by a larger effective value of U. The larger surface recombination speed pulls more of the recombination current from the bulk to the surface where it has higher fractional noise. The larger potential jump U

increases the applicable Hooge parameter according to Eq. (121). Finally, the larger fractional 1/f noise levels of ion implanted junctions is mainly caused by the 1-2 orders of magnitude lower carrier lifetimes in Eqs. (118)-(132), which yield 1-2 orders of magnitude smaller N_{eff} values and larger fractional noise power values by the same factor.

In order to reduce the fractional noise level, our theory suggests the use of a surface passivation which lowers the surface recombination speed and the surface potential jump U. The ideal "surface" would be a gradual increase of the gap width starting from the bulk through compositional changes leading to a completely insulating stable surface outwards, without the generation of surface recombination centers. In addition, the life time of the carriers should be kept high, and abrupt or pinched regions in the junction should be avoided. The reasonable choice of other junction parameters, including the steepness of the junction and the geometry should yield lower injection - extraction and bulk recombination noises by emphasizing the presence of the larger hole masses in the denominators of the above expressions. Finally, coherent state 1/f noise should be avoided by all means by optimizing the dimensions, i.e., keeping them sufficiently small for individual devices.

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III.4 QUANTUM 1/f NOISE IN MIS INFRARED DETECTORS

A. INTRODUCTION

The quantum 1/f noise theory is applied here to the calculation of 1/f noise in the various dark current components which limit the performance of MIS infrared detectors; the photo-current carries no quantum 1/f noise, because the elementary process of photo-generation of an electron - hole pair is almost free of infrared radiative corrections, and later-on the carriers are collected anyway. The dark diffusion current transports minority carriers from the bulk to the surface inversion layer and is affected by coherent states quantum 1/f noise in the mobility, as long as the device is larger than approximately 10 μm^2 in area. For smaller devices we expect the smaller conventional quantum 1/f noise of the scattering cross sections to be expressed in the mobility and diffusion fluctuations. Quantum 1/f recombination speed fluctuations appear as fluctuations in the thermally generated current of minority carriers both at the surface and in the depletion region, which are majority carriers in the inversion layer. The rate of tunneling also presents quantum 1/f fluctuations which are calculated both for band to band tunneling and for tunneling via bandgap states, with the help of the same fundamental quantum 1/f formula used for the diffusion and recombination currents. Conventional quantum 1/f contributions are smaller for holes than for electrons, because they are inversely proportional to the effective mass.

B. CURRENTS IN MIS DETECTOR STRUCTURES

MIS detectors[35] are different from photovoltaic (or pn junction) detectors, because they do not contain a pn junction obtained by inhomogeneous doping, and use an insulated field plate, or gate, placed on top of a homogeneously doped narrow-bandgap semiconductor. The gate is used to control the surface potential, driving the semiconductor surface into deep inversion. The field of the induced quasi-pn junction obtained under the surface of the gate in the homogeneous semiconductor material is used to separate the carriers generated by photo-electrically induced band to band transitions just as in a photovoltaic device. The MIS infrared

detector is therefore similar to a capacitively coupled photovoltaic detector, without the inconvenience of inhomogeneous doping processes.

MIS detectors are operated in the pulsed regime by applying the gate potential which creates the inversion under the surface for a finite time only, and by applying subsequently a potential which flattens the energy bands near the surface and releases the carriers which had accumulated from photo-electric effect and dark current processes during the preceding interval. The electrical signal obtained when the carriers are released, i.e., during readout, is proportional to the number of carriers accumulated, and therefore to the total current supplying the inverted volume under the surface with minority carriers from the bulk and from various thermal and photo-electric processes in the depletion, inversion and surface regions. This electrical signal is used in order to determine the flux of infrared radiation. For this determination, however, the dark current contribution needs to be subtracted first.

The dark current is the current supplying the potential well, defined by the inversion region under the surface, with minority carriers in the absence of the applied infrared flux. Any low - frequency fluctuation in the dark current will be interpreted as a fluctuation in the major infrared flux signal. Therefore fluctuations of the dark current at frequencies below the readout frequency will limit the performance of infrared detectors. In the pulsed mode of operation considered here, the dark current is monitored only during the inverted phase, when carriers are accumulated in the potential well. Therefore the cross sections and process rates which control the intensity of the dark current are not sampled continuously by us either. Nevertheless, the quantum 1/f fluctuations of these cross sections and process rates will be practically the same as if we would have observed them continuously. Indeed, we first note that the changes caused by the applied gate voltages in the microscopic flux of electrons testing all cross sections and process rates defined in a semiconductor do not affect the cross sections or process rates themselves. There is a similar flux interrogating the cross sections in thermal equilibrium. How much and in which way this flux is modified by the applied voltage is not important; there is always some flux there to test the cross sections at all energies and scattering angles. Being the

ratio of the scattered current to the incoming flux, the cross section is not affected by flux changes. Second, we note that since it contains the number of carriers N in the denominator, quantum 1/f noise should be dependent in general on the flux level used in testing the cross sections or process rates. However, N is not the number of carriers accumulated in the well during the pulse of the gate voltage. N is the number of carriers used to define the scattered, or outgoing current at any time during the pulse of the gate voltage, i.e., the number of carriers effectively carrying the scattered current in stationary scattering conditions. The stationary scattering conditions are established in a time of the order of the mean carrier collision time, which is much shorter than the pulse length of the gate voltage. Therefore, the value of N during pulsed operation is not dependent on the pulse length, and is not different from the value corresponding to an infinite pulse length or continuous (cw) operation, even if the applied voltage is so large that the microscopic flux interrogating the cross sections differs considerably from the thermal equilibrium value. We conclude that, indeed, the quantum 1/f noise affecting the cross sections and process rates is not changed by the pulsed operation. This independence of 1/f noise on the continuous or discontinuous character of any applied bias has been experimentally verified[38] during the last 2 decades, and has been found to be in agreement with the interpretation of 1/f noise in terms of fundamental resistance fluctuations. Although the experimental verification was performed on fluctuations in conduction only (involving mainly scattering cross sections), from the above discussion based on the concept of quantum 1/f noise we know that the similarity of quantum 1/f noise in the continuous and pulsed regimes should be also true for quantum 1/f fluctuations in recombination cross sections and tunneling rates.

The dark current has to be subtracted from the total current in a (HgCd)Te MIS device to yield the photocurrent. Therefore, the minority - carrier dark current is the single most important parameter for the operation of MIS devices as detectors of infrared radiation[35]. This applies both to operation of MIS devices in the thermal equilibrium mode, in which the dark current determines the MIS diode impedance, and to operation in the dynamic, or integrating mode, in which the gate voltage is

pulsed, and in which the minority - carrier dark current determines the storage time of the device. The main component of the dark current in narrow-bandgap HgCdTe is the tunneling current via bandgap states[35], which can also be considered as an electric breakdown effect. In general, the tunneling current occurs both through band to band transitions and through intermediary states. The band to band tunneling current through a simple triangular barrier is

$$I_{tb} = (q^3 E \Phi_s / 4\pi^3 \hbar^2)(2m^*/E_g)^{1/2} \exp[4(2m^*)^{1/2} E_g{}^{3/2} / 3q\hbar E], \quad (134)$$

where E is the electric field associated with the barrier, and E_g is the band gap. The electric field can be approximated by the electric field at the semiconductor surface

$$E = (2qn_o \Phi_s / \varepsilon\varepsilon_o)^{1/2}, \quad (135)$$

where Φ_s represents the empty well surface potential, and n_o is the doping concentration. Substituting this value into Eq. (135), with $m^*/m_o = 7\ 10^{-2}\ E_g$, we obtain[35]

$$I_{tb} = 10^{-2} n_o{}^{1/2} \Phi_s{}^{3/2} \exp[-\ 4.3\ 10^{10} E_g{}^2 / (n_o \Phi_s)^{1/2}]\ \mathrm{A/cm^2}, \quad (136)$$

where n_o is in cm^{-3} and E_g, Φ_s in volts. Therefore the tunneling current is strongly dependent on the bandgap and also depends on the doping concentration and the surface potential.

Experimental values of the tunneling current are usually larger than Eq. (136) because of the additional effect of tunneling via bandgap states. This effect is particularly important in n - type devices[39]. Indeed, in n - type devices the applied gate voltage is negative in order to produce depletion at the surface. The energy bands are therefore curved upwards at the surface, and transitions of electrons from the valence band to Shockley - Read (SR) states at the middle of the band gap, as well as the subsequent transitions from these states to the conduction band are facilitated by the presence of many defects right at the surface of the semiconductor. In p - type devices the similar indirect tunneling

processes occur farther away from the surface, because in this case the bands are curved downwards at the surface, and transitions of electrons from the valence band to the centers at the middle of the band gap, as well as the transitions from the centers to the conduction band well at the surface, occur right where the curvature begins, i.e., further away from the surface. We conclude that in p - type devices there will be fewer SR centers active in indirect tunneling, and therefore the tunneling current I_c via SR centers at a given temperature and a given applied gate voltage will be smaller. The tunneling current will be further reduced in p - type devices due to the lower density of states present in the surface potential well due to quantization of the motion of the electrons in the potential well at the surface. The reduced values of the dark current in p - type devices correspond to higher values of the breakdown field in these devices. The best measured value[39] of the breakdown field in 10 μm cutoff p - type devices is in excess of 1.0 V/m, whereas that for n - type material of similar bulk defect quality is 0.5 V/m. On the other hand, the minority carriers diffusion current is larger in p - type devices due to the smaller mass and higher diffusion constant of electrons compared to holes. *The advantage of p - type devices is therefore considerable only in* the case of very narrow band gap and very long cutoff wavelengths. We shall therefore consider both the case of p - type and n - type devices. The large diffusion current present in p - type devices corresponds to the large value of the diffusion length of electrons and can be reduced by thinning the device, i.e., by reducing its thickness well below the diffusion length.

In general the dark current I can be written in the form

$$I = I_d + I_r + I_s + I_{tb} + I_{tc} + I_{tsc} + I_b + q\eta\Phi. \tag{137}$$

The eight terms on the right hand side correspond to minority carrier diffusion from the bulk, generation-recombination from SR centers in the depletion region, generation from SR centers at the surface, band to band tunneling, tunneling via SR centers, tunneling via surface centers, recombination on the back surface and photoelectric generation with efficiency η by the thermal radiation background flux Φ. We shall give the

formulae[39] which determine these terms below, also including an example of their calculation in a p - type device

$$I_d = (qn_i^2/p_o)[kT\mu_n/q\tau_n]^{1/2} = (1.6\ 10^{-19}C\ 36\ 10^{24}\ cm^{-6}/10^{15}cm^{-3})$$
$$[0.01V\ \ 1.5\ \ 10^5(cm^2/Vs)/10^{-6}s]^{1/2} = 2\ 10^{-4}A/cm^2, \qquad (138)$$

$$I_r = qn_iW/2\tau_o = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ 2\ 10^{-4}cm\ /(2\ 10^{-6}s)]$$
$$= 10^{-4}A/cm^2 \qquad (139)$$

$$I_s\ (\approx I_b\ ) = qn_is/2 = [(1.6\ 10^{-19}C\ 6\ 10^{12}cm^{-3}\ \ 20cm/s]$$
$$= 1.8\ 10^{-5}A/cm^2, \qquad (140)$$

$$I_{tc} = 10^{-13}(N_r\Phi_s/E_g)\exp[-5.3\ \ 10^6E_g{}^2/E] = 10^{-13}(4\ \ 10^{12}cm^{-3}0.2V/0.062V)$$
$$\exp[-5.3\ \ 10^6(0.062V)^2/3000V/cm] = 8\ \ 10^{-4}\ A/cm^2, \qquad (141)$$

and I_{tb} was given by Eq. (136), yielding $10^{-7}A/cm^2$. Here $n_i = 6\ 10^{12}cm^3$ is the intrinsic carrier concentration, and the concentration of holes was taken to be $p_o = 10^{15}cm^{-3}$. The mobility $\mu = 1.5\ 10^{15}cm^2/Vs$ as well as the life time $\tau_n = 10^{-6}s$ of the minority carriers have to be replaced by μ_p and τ_p for the case of n - type devices. The surface recombination speed was denoted by s = 40cm/s, and the concentration of intermediary states effective for tunneling was denoted by $N_r = 4\ 10^{12}cm^{-3}$. The band gap considered was $E_g = 0.065eV$, the surface potential $\Phi_s = 0.2V$ and the electric field below the surface E = 3000V/cm. All numerical values have been included only as an example and are not characteristic of a particular device. The numerical factors in Eq. (141) correspond to p - type HgCdTe with a 20 μm cutoff wavelength and were taken from the paper of Kinch and Beck[39]. For n - type devices we also need to include tunneling via surface states. If the density of fast surface states is denoted by $N_{fs} = 10^{12}\ cm^{-2}V^{-1}$, the current generated from tunneling via a uniform density of fast surface states across the band gap will be given by[35]

$$I_{tsc} = -qp_1N_{fs}kT/2q = 1.75\ 10^{-4}\ A/cm^2. \qquad (142)$$

The sum of the first seven currents on the right hand side of Eq. (137) must be smaller than the eighth which corresponds to the thermal radiation background flux, for background limited (BLIP) performance. Although not all terms in the dark current are of importance, we still retain them at this point, because their quantum 1/f noise may be quite significant, even if the corresponding current is negligible. We shall now proceed with the calculation of quantum 1/f noise contributions from all these currents.

C. QUANTUM 1/f NOISE SOURCES

1. 1/f Noise in the Diffusion Current

The diffusion limited dark current I_d will exhibit 1/f noise due to conventional quantum 1/f fluctuations in the scattering cross sections of the carriers due to phonons and impurities. We apply the fundamental formula given by Eq. (3) for an individual scattering process in which the velocity change v is given by the thermal energy of the carriers, with the assumption that the collisions are perfectly randomizing collisions. If the velocity v is rotated by an angle θ in an elastic collision, the velocity change magnitude is $|\Delta v| = 2v \sin(\theta/2)$. Averaging over all angles and velocities, we obtain

$$\Delta v^2 = 4v^2 \sin^2(\theta/2) = 2v^2, \tag{143}$$

and therefore from Eq. (1.1) we get in thermal equilibrium at the temperature T the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)(6kT/m^*c^2), \tag{144}$$

where we have assumed a Maxwell distribution of velocities. For $Hg_{1-x}Cd_xTe$ with $x = 0.2$ we have $m_n^* = 0.008m_o$ and for $x = 0.3$ we have $m_n^* = 0.02m_o$. Therefore we obtain $\alpha_H = 2\ 10^{-7}$ in the first case and $\alpha_H = 7.5\ 10^{-8}$ in the second case.

For the case of Umklapp scattering, which occurs in semiconductors only to a limited extent due to the relatively small number of high momentum phonons available at the temperature T, the momentum change of the electron is given by the smallest reciprocal lattice vector, and therefore $|\Delta v| = h/am^*$. We therefore obtain the quantum 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)(h/m^*ac)^2, \tag{145}$$

which is much larger than Eq. (144), but has to be multiplied with a negative exponential which describes the scarcity of phonons with momentum of the order of a reciprocal lattice vector. [The negative exponential $e^{-\Theta/T}$ could be included in the current weight factor which will be defined below in Eq. (), but we prefer to include it here already]. Combining Eqs. (144) and (145), we obtain for conventional 1/f noise in the mobility and diffusion coefficients

$$\alpha_H = (4\alpha/3\pi)[(6kT/m^*c^2) + (h/m^*ac)^2 \exp(-\Theta/T)], \tag{146}$$

where Θ is about half the Debye temperature for simple metals, but may be higher, of the order of the Debye temperature, for semiconductors.

The quantum 1/f noise considered so far is known as conventional quantum 1/f noise, and affects cross sections and process rates. In sufficiently large semiconductors samples we expect coherent state quantum 1/f noise. For this type the 1/f noise coefficient is given by

$$\alpha_{coh} = 2\alpha/\pi = 4.6\ 10^{-3}. \tag{147}$$

The values of the quantum 1/f noise coefficient given by Eqs. (143) - (147) can be used to calculate the quantum 1/f noise which affects the various currents listed in Eq. (137). We first consider the case of the dark diffusion current of electrons from the bulk through the surface barrier in a p -type MIS device, similar to diffusion in a n^+p junction, because in both cases the current is determined by the diffusion of electrons which are minority carriers, against the built - in field of a Boltzmann potential

barrier into the surface well, and by the thermal generation of carriers there. We start with the derivation of the mobility fluctuation part of quantum 1/f noise in a n^+-p diode. For the MIS barrier, just as for a diffusion limited n^+-p junction, the current is controlled by diffusion of electrons into the p - region over a distance of the order of the diffusion length $L = (D_n \tau_n)^{1/2}$ which is usually shorter than the length w_p of the p - region. If N(x) is the number of electrons per unit length and D_n their diffusion constant, the electron current at x is

$$I_{nd} = - eD_n dN/dx, \tag{148}$$

where we have assumed a planar junction and taken the origin x = 0 in the junction plane. Diffusion constant fluctuations, given by kT/e times the mobility fluctuations, will lead to local current fluctuations in the interval Δx

$$\delta\Delta I_{nd}(x,t) = I_{nd}\Delta x \delta D_n(x,t)/D_n. \tag{149}$$

The normalized weight with which these local fluctuations representative of the interval Δx contribute to the total current I_d through the diode at x = 0 is determined by the appropriate Green function and can be shown to be (1/L)exp(-x/L) for $w_p/L >> 1$. Therefore the contribution of the section Δx is

$$\delta\Delta I_d(x,t) = (\Delta x/L)\exp(-x/L) I_{nd} \delta D_n(x,t)/D_n, \tag{150}$$

with the spectral density

$$S_{\Delta I_d}(x,f) = (\Delta x/L)^2 \exp(-2x/L)\ I_{nd}^2\ S_{D_n}(x,f)/D_n^2. \tag{151}$$

For mobility and diffusion fluctuations the fractional spectral density is given by α_{Hnd}/fNx, where α_{Hnd} is determined from quantum 1/f theory according to Eqs. (143) -(147). With Eq. (148) we obtain then

$$S_{\Delta I_d}(x,f) = (\Delta x/L^2)\ \exp(-2x/L)\ (eD_n dN/dx)^2\ \alpha_{Hnd}/fN. \tag{152}$$

The electrons are distributed according to the solution of the diffusion equation, i.e.

$$N(x) = [N(0) - N_p] \exp(-x/L) + N_p; \quad dN/dx = -\{[N(0) - N_p]/L\} \exp(-x/L). \tag{153}$$

Substituting into Eq. (152) and simply summing over the uncorrelated contributions of all intervals Δx, we obtain

$$S_{Id}(f) = \alpha_{Hnd}(eD_n/L^2)^2 \int_0^{w_p} [N(0) - N_p]^2 e^{-4x/L} dx \,/\{[N(0) - N_p]e^{-x/L} + N_p\}. \tag{154}$$

We note that $(eD_n/L^2) = (e/\tau_n)$. With the expression of the saturation current $I_0 = e(D_n/_n)^{1/2}N_p$ and of the current $I = I_0[\exp(eV/kT) - 1]$ we can carry out the integration

$$S_{Id}(f) = \alpha_{Hnd}(eI/f\tau_n) \int_0^1 a^2u^3 du/(au + 1)$$

$$= \alpha_{Hnd}(eI/f\tau_n)\ [F(a) - F(a\ e^{-w_p/L})] \approx \alpha_{Hnd}(eI/f\tau_n)aw_p/[(a + 1)L], \tag{155}$$

the last form being for $w_p << L$. Here F(a), a and u are given by Eq. (93). For $w_p >> L$ we have F(o) = 0, and the second term in rectangular brackets drops out in Eq. (155).

Eq. (155) only contains the fluctuations in the mobility and the diffusion constant. In a similar way we calculate the quantum 1/f fluctuations of the recombination rate in the bulk of the p - region. We have for the recombination current $\Delta I_R(x)$ in a section Δx, if N'(x) is the excess carrier density,

$$\Delta I_R(x) = eN'(x)\Delta x/\tau_n \tag{156}$$

Putting $C_n = 1/\tau_n$ and bearing in mind that τ_n, and hence C_n, fluctuates, we have for the section Δx,

$$\delta\Delta I_R(x, t) = \Delta I_R(x)[\delta C_n/C_n] \tag{157}$$

with

$$S_{C_n}(x,f)/C_n^2 = \alpha_{hnr}/fN, \tag{158}$$

so that, since $N(x) = N/\Delta x$,

$$S_{\Delta I_R}(x,\ f) = \Delta I_R^2(x)[\alpha_{Hnr}/fN] = \alpha_{Hnr}\{e^2[N'(x)]^2/f\tau_n^2 N(x)\}\Delta x \tag{159}$$

where $N'(x) = [N(0) - N_p]\exp(-x/L)$ and $N(x) = N'(x) + N_p$ as before.

It is easily shown that the fluctuating current $\Delta I(x,\ t)$ at the junction is

$$\Delta I(x,\ t) = \Delta I_R(x,\ t)\exp(-x/L), \tag{160}$$

so

$$S_I(f) = \alpha_{Hnr}(e^2 N_p L/f\tau_n^2)\int_0^{w_p}\{[N'(x)]^2/N_p N(x)\}\exp(-2x/L)d(x/L)$$

$$= \alpha_{Hnr}(eI_0/f\tau_n)\int_0^1[a^2u^3/(au + 1)]du = \alpha_{Hnr}[eI/f\tau_n][F(a) - F(ae^{-w_p/L})], \tag{161}$$

where F(a), a, and u have the same meaning as before.

We can use the similarity of the quantum 1/f noise results for diffusion current fluctuations caused by mobility fluctuations and by recombination speed fluctuations in order to combine both into a single formula

$$S_I(f) = (\alpha_{Hd} + \alpha_{Hnr})[eI/f\tau_n][F(a) - F(ae^{-w_p/L})]. \tag{162}$$

In the limit of very short devices ($w_p << L$) the last factor becomes $aw_p/[(a + 1)L]$, and in the limit of long MIS devices ($w_p >> L$) it simply becomes F(a). In addition we have a current noise contribution S_{Ib} from the quantum 1/f fluctuation of the recombination speed s on the back surface.

So far we have considered only conventional quantum 1/f noise which is applicable to sufficiently small devices. In general, however, we must interpolate between conventional and coherent quantum 1/f noise, according to the relation

$$\alpha_H = [1/(1+s)][2\alpha A/f] + [s/(1+s)][2\alpha/\pi f], \quad (163)$$

where $s = E_m/E_k = 2e^2N'/m^*c^2$, and $N' = nS$ is the number of the carriers per unit length of the device. Here s represents the ratio between the magnetic energy per unit length and the kinetic energy per unit length of the device. The quantity $e^2/m^*c^2 = r_0 m/m^*$ can be calculated in terms of the classical radius of the electron $r_0 = 2.84\ 10^{-13}$ cm. Then we obtain s $= 2\ r_0N'm/m^*$, i.e., the parameter s represents twice the number of carriers present in a length of the device equal to the classical radius of the electron. We must compare s with A, and if s << A we apply conventional quantum 1/f noise, whereas for s >> A we have to apply coherent quantum 1/f noise. In general the approximate formula of Eq. (163) must be used for the transition region.

The dimensionless parameter s is easy to calculate in any practical case. For instance in the case of a MIS device of area 50 μm x 50 μm with a concentration of carriers of 10^{16} cm^{-3} we obtain $N' = 2.5\ 10^{11}$/cm, and with $m/m^* = 50$ we obtain s = 7. On the other hand, we can estimate A for conventional quantum 1/f noise and we will certainly find A << 1, because the velocity change of the carriers must be much smaller than the speed of light. Therefore, in this case we must apply coherent quantum 1/f noise, because s >> A. Consequently, in Eq. (3.20) we must set

$$\alpha_{Hd} + \alpha_{Hnr} = \alpha_{coh} = 4.6\ 10^{-3} \quad (164)$$

The coherent state quantum 1/f noise coefficient thus replaces the total conventional Hooge parameter.

2. 1/f Noise of the Recombination Current Generated in the Depletion Region

The quantum 1/f noise of the recombination current thermally generated in the depletion region arises from quantum 1/f fluctuations of the bulk recombination rates in the depletion region. The difference between the recombination rate R and the generation rate G is given by

$$R(x) - G(x) = [pn - n^2_i]/[(n + n_1)\tau_{po} + (p + p_1)\tau_{no}] \tag{165}$$

where n_1 and p_1 are electron and hole densities when the Fermi level lies at the trap level. If the trap level lies at the intrinsic level, $n_1 \approx p_1 \approx n_i$. Moreover, τ_{po} and τ_{no} are time constants for electrons and holes. If A is the cross - sectional area of the junction, the current is

$$I_r = e\int_0^w [R(x) - G(x)]A\,dx = e\int_0^w [pn - n^2_i]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]A\,dx, \tag{166}$$

where w is the width of the space - charge region and the trap level is assumed to lie at the intrinsic level.

We now turn to the g - r noise. The time constants τ_{po} and τ_{no} fluctuate in a 1/f fashion due to the quantum 1/f fluctuations in the recombination cross sections, and this produces the quantum 1/f contribution to g - r noise. We now write

$$\tau_{no} = 1/C_n; \quad \tau_{po} = 1/C_p, \tag{167}$$

where C_n and C_p are the generation (or combination) rates for a single electron and for a single hole, respectively. Consequently

$$\delta\tau_{no}/\tau_{no} = -(\delta C_n/C_n); \quad \delta\tau_{po}/\tau_{po} = -(\delta C_p/C_p) \tag{168}$$

We now apply this knowledge to Eq. (165) and observe that

$$\delta(R - G) = [R(x) - G(x)]\times$$
$$\{[(n + n_i)\tau_{po}(\delta C_p/C_p)] + [(p + n_i)\tau_{no}(\delta C_n/C_n)]\}/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]. \tag{169}$$

so that with

$$\delta I_r = e\int_0^w \delta[R(x) - G(x)]A dx, \tag{170}$$

the noise is

$$S_{I_r}(f) = e^2 \int_0^w \int_0^w [R(x) - G(x)]A[(R(x') - G(x')]A$$
$$\{[(n + n_i)^2 \tau_{po}{}^2 S_{Cp}(x, x', f)/C_p{}^2 + (p + n_i)^2 \tau_{no}{}^2 S_{Cn}(x, x', f)/C_n{}^2]$$
$$/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2\}dxdx', \tag{171}$$

since C_p and C_n are independent.

We now observe that

$$pn - n^2{}_i = n^2{}_i[\exp(eV/2kT) - 1][\exp(eV/2kT) + 1], \tag{172}$$

and that the integrand in Eq. (171) has an appreciable value only if $p \approx n \approx n_i \exp(eV/2kT)$. By substituting $n + n_i \approx p + n_i \approx n_i[\exp(eV/2kT) + 1]$ we define an effective width w_{eff} such that

$$eA \int_0^w [(pn - n^2{}_i)dx]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]$$
$$= eA\{(pn - n^2{}_i)/n_i[\exp(eV/2kT) + 1]\}[w_{eff}/(\tau_{po} + \tau_{no})]. \tag{173}$$

We may thus write

$$I_r = I_{gr} = eAw_{eff}n_i[\exp(eV/2kT) - 1]/(\tau_{po} + \tau_{no})$$
$$= [eN_{eff}/(\tau_{po} + \tau_{no})]\tanh eV/2kT, \tag{174}$$

where $N_{eff} = Aw_{eff}n_i[\exp(eV/2kT) + 1]$ is the effective number of hole - electron pairs taking part in the conduction and noise processes. This equation is exact but not very useful since it contains the unknown parameter w_{eff}.

We now turn to Eq. (171) and observe that

$$S_{Cp}(x, x', f)/C^2{}_p = \{(\alpha_{Hp}/f)/[R(x') + G(x')](\tau_{po} + \tau_{no})A\}\delta(x' - x), \tag{175}$$

and

$$S_{C_n}(x, x', f)/C^2{}_n = \{(\alpha_{Hn}/f)/[R(x') + G(x')](\tau_{po} + \tau_{no})A\}\delta(x' - x). \quad (176)$$

The factor $(\tau_{no} + \tau_{po})$ comes in because $S_{C_p}(x, x', f)/C_p{}^2$ and $S_{C_n}(x, x', f)/C_n{}^2$ must be independent of τ_{po} and τ_{no} if $p \approx n \approx n_i \exp(eV/2kT)$. This yields, if we integrate over the δ function

$$S_{I_r}(f) = e^2 \int_0^W \{[R(x) - G(x)]^2 A/(\tau_{po} + \tau_{no})[R(x) + G(x)]\}$$
$$\{[(n + n_i)^2\tau_{po}{}^2\alpha_{Hp}/f + (p + n_i)^2\tau_{no}{}^2\alpha_{Hn}/f]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2\}dx \quad (177)$$

We now observe that the second factor in Eq. (177) is practically a constant as long as p and n are comparable. We may thus bring that factor outside the integral sign and write

$$[(n + n_i)^2\tau_{po}{}^2\alpha_{Hp}/f + (p + n_i)^2\tau_{no}{}^2\alpha_{Hn}/f]/[(n + n_i)\tau_{po} + (p + n_i)\tau_{no}]^2 = \alpha_H/f, \quad (178)$$

where α_H is given by

$$\alpha_H = [\tau_{po}/(\tau_{po} + \tau_{no})]^2\alpha_{Hp} + [\tau_{no}/(\tau_{po} + \tau_{no})]^2\alpha_{Hn} \quad (179)$$

We thus have

$$S_{I_r}(f) = [\alpha_H e/f(\tau_{no} + \tau_{po})]e \int_0^W [R(x) - G(x)]^2 A/[R(x) + G(x)]dx$$
$$= [\alpha_H e I_{gr}/f(\tau_{no} + \tau_{po})]\tanh [eV/2kT]. \quad (180)$$

We can now prove Eq. (180) in a Hooge - type formulation. Here we put

$$S_{I_r}(f)/I_r{}^2 = \alpha_H/fN_{eff} \quad (181)$$

But according to Eq. (174), $I_{gr} = (eN_{eff}/\tau)\tanh eV/2kT$, so that

$$S_I(f) = \alpha_H[eI_r/f\tau]\tanh eV/2kT, \qquad \tau = (\tau_{no} + \tau_{po}) \quad (182)$$

in agreement with Eq. (180).

We can also prove Eq. (180) from the following consideration. We write $I_r = (N_{eff}I')\tanh eV/2kT$, where $I' = e/\tau$ fluctuates. We then have

$$S_{I'}(f)/I'^2 = \alpha_H/f \text{ or } S_{I'}(f) = \alpha_H/f(e/\tau)I \tag{183}$$

so that, since the N_{eff} hole - electron pairs are independent

$$S_I(f) = N_{eff}S_{I'}(f) = (eN_{eff}I'/f\tau)\alpha_H = [(eI_{gr}/f\tau)\alpha_H]\tanh eV/2kT, \tag{184}$$

where α_H is given by Eq.(179) and $\tau = (\tau_{no} + \tau_{po})$.

The last two approaches are easily extended to other cases; the method works as long as a time constant τ and an N_{eff} can be defined.

We finally evaluate α_{Hp} and α_{Hn} from quantum 1/f noise considerations:

$$\alpha_{Hn} = (4\alpha/3\pi)(\Delta v^2{}_n/c^2) = (4\alpha/3\pi)\{[2ea(V_{dif} - V) + 3kT]/m^*{}_n c^2\}, \tag{185}$$

$$\alpha_{Hp} = (4\alpha/3\pi)(\Delta v^2{}_p/c^2) = (4\alpha/3\pi)|[2e(1 - a)(V_{dif} - V) + 3kT]/m^*{}_p c^2|, \tag{186}$$

and as a consequence (see Eq. 179),

$$\alpha_H = (4\alpha/3\pi)[2e(V_{dif} - V) + 6kT]/[(m^*{}_n)^{1/2} + (m^*{}_p)^{1/2}]^2c^2 \tag{187}$$

The problem has hereby been solved. Note that in $Hg_{1-x}CD_xTe$ with $x = 0.3$, $m^*{}_n = 0.02$ m, $m^*{}_p = 0.55$ m, so that $[(m^*{}_n)^{1/2} + (m^*{}_p)^{1/2}]^2 = 0.78$ m, very much larger than $m^*{}_n$.

3. Noise in the Surface Recombination Current

The surface recombination current is a dark current component originating from surface states at the interface between the surface passivation layer and the bulk. The recombination cross sections of these states will exhibit quantum 1/f noise. The quantum 1/f noise coefficient α_H reflects the velocity change of carriers involved in this process

$$\alpha_H = (4\alpha/3\pi)(2/m^*c^2)[3kT/2 + eU/2 + 0.1eV], \qquad (188)$$

where U is the surface potential jump present between the surface passivation layer and the bulk even if no gate voltage is applied, and V is the applied gate voltage which we take with a coefficient less than unity, here for example with a weight of 0.1.

The calculation of surface recombination quantum 1/f noise is similar to the calculation of quantum 1/f noise from recombination in the space charge region. However, in this case the cross sections are not distributed over the width of the junction, but rather are concentrated at the surface. Therefore we can repeat the above derivation and we obtain an expression of the form

$$S_{Is}(f) = \alpha_H e I_s[\tanh x] / f(\tau_{no} + \tau_{po}). \qquad (x = eV/2kT) \qquad (189)$$

We note that the fabrication process of MIS structure introduces less bulk defects than the fabrication of photovoltaic devices. Nevertheless, the fabrication of MIS devices introduces some defects in the bulk layer located right under the surface. These defects will manifest themselves through a contribution to indirect tunneling.

4. Quantum 1/f Noise in the Tunneling Rate

In the case of tunneling from a surface accumulation layer to the bulk, the velocity change of the carriers will lead us from the thermal velocity of the carrier on one side of the barrier to the thermal velocity of a carrier on the other side of the barrier, if band to band tunneling is considered. If, however, the tunneling goes via intermediate states located in the bandgap, the velocity is zero as long as the carrier is stationary in the intermediate state. We can therefore write the 1/f noise coefficient

$$\alpha_H = (4\alpha/3\pi)6kT/m^*c^2 \qquad (190)$$

for band to band tunneling, and

$$\alpha_H = (4\alpha/3\pi)3kT/m^*c^2 \tag{191}$$

for tunneling via intermediate states in the band gap, where we have considered the average squared velocity change two times smaller. The effective mass is the mass of the minority carriers in the bulk material.

For band to band tunneling from a surface inversion layer to the bulk, the velocity change of carriers corresponds to an energy difference of the order of the band gap E_g plus an energy difference of the order of the thermal energy 3kT/2, provided we are dealing with deep inversion, as used in practical MIS devices in the pulsed mode of operation. This yields the quantum 1/f coefficient

$$\alpha_H = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 \tag{192}$$

for band to band tunneling. For tunneling via intermediate states in the band gap the corresponding energy difference will be smaller, and therefore we replace E_g by $E_g/2$

$$\alpha_H = (4\alpha/3\pi)(E_g/2 + 3kT/2)m^*c^2 \qquad (193).$$

This relation is applicable both if the intermediate states are located in the depletion region or at the surface.

In the last four equations we did not divide by the number of carriers simultaneously involved in the tunneling process, because this number is less than unity for practical tunneling currents. Whenever a cross section or a process rate is tested with one electron or less than one electron at a time, the effective number of electrons in the denominator of the quantum 1/f formula must be replaced by unity. Let us calculate the average number of carriers simultaneously present in the tunneling process at any time. The tunneling process occurs over a distance $d = E_g/eE$, and the speed v of the carriers will be of the order of the thermal speed in the case of an accumulation layer, and of the order of the band gap energy in

the case of a surface inversion layer. Dividing d by v, we obtain a tunneling time $t = 6\ 10^{-13}$s for accumulation layers and $t = 3\ 10^{-13}$s for inversion layers. From Eq. (141) we know that the tunneling current is of the order of 10^{-3}A/cm^2. Multiplying this by t, we obtain 3 - 6 10^{-16} C, i.e., 2000 - 3000 electrons/cm^2 tunneling simultaneously in a device of 1 cm^2 area. In a device of dimensions 50 µm x µ50 m = 2.5 10^{-5} cm^2 the average number of carriers in the process of tunneling at any time is therefore 0.05 - 0.075, and this is indeed much less than unity. Nevertheless, if the area of the device exceeds 5 10^{-4} cm^2, Eqs. (190) - (193) require an additional factor e/tI_tA which makes the noise spectral density proportional to I_t and A, rather than to the square of these quantities.

The photoelectric current will reflect the fluctuations in the number of photons arriving from the radiation background. The quantum efficiency will not exhibit considerable quantum 1/f noise, because the generated carriers will be corrected with certainty. Therefore the collection of photoelectrically generated carriers is not controlled by any cross section or process rate affected by considerable quantum 1/f noise. If we neglect the 1/f noise generated *in the series resistance of the diode, there should* be no photoelectrically generated 1/f noise from a short - circuited diode.

Since the various dark current fluctuations with 1/f spectrum are statistically independent, the total 1/f noise is simply obtained by summing all contributions.

D. 1/f NOISE LIMITED PERFORMANCE OF MIS DIODES

From Eq. (2.4) we write the total dark current fluctuation in the form

$$\delta I_d = \delta I_d + \delta I_r + \delta I_s + \delta I_{tb} + \delta I_{tc} + \delta I_{tsc} + \delta(q\eta\Phi), \tag{194}$$

and the spectral density of current fluctuations will be neglecting $\delta(q\eta\Phi)$,

$$S_{Id} = S_{Id} + S_{Ir} + S_{Is} + S_{Itb} + S_{Itc} + S_{Itsc}. \tag{195}$$

Here we have lumped the recombination current on the back surface I_b together with the surface recombination (generation) current I_s. If we denote all the corresponding spectral densities of fractional fluctuations by a prime, $S'_{Ii} = S_{Ii}/I_i^2$, we obtain

$$S'_{Id} = (I_{dif}/I_d)^2 S'_{dif} + (I_{dep}/I_d)^2 S'_{Idep} + (I_s/I_d)^2 S'_{Is} + (I_{tb}/I_d)^2 S'_{Itb} + (I_{tc}/I_d)^2 S'_{Itc} + (I_{tsc}/I_d)^2 S'_{Itsc}. \quad (196)$$

This equation was obtained by dividing the previous equation through I_d^2, and shows that the biggest contribution will not necessarily come from the process with the highest fractional quantum 1/f noise, i. e., with the highest 1/f noise coefficient. The weight of each type of noise is determined by the corresponding squared current ratio.

The detectivity of infrared detectors is limited in general by three types of noise: (i) current noise in the detector, (ii) noise due to background photons (photon noise), (iii) noise in the electronic system following the detector. We shall neglect here the background photon noise and the noise in the electronic system. The detectivity is defined as

$$D^*(\lambda,f) = (A\Delta f)^{1/2}/NEP(\mathrm{cmHz}^{1/2}/W) \quad (197)$$

where A is the area of the detector, NEP the noise equivalent power defined as the r.m.s. optical signal of wavelength required to produce r.m.s. noise voltage (current) equal to the r.m.s. noise voltage (current) in a bandwidth Δf, and f is the frequency of modulation. The noise equivalent power NEP is given by

$$NEP = (h\nu/\eta q)(S_{Id}(f)\Delta f)^{1/2}. \quad (198)$$

Therefore we obtain for the detectivity

$$D^*(\lambda,f) = (q\eta\lambda/hc)[A/S_{Id}(f)]^{1/2} = (q\lambda/hc)[S_{Id}(f)]^{-1/2} \quad (199)$$

We notice that $D^*(\lambda,f)$ is proportional to λ up to the peak wavelength λ_c. For $\lambda > \lambda_c$ we have $\eta = 0$ and thus $D^*(\lambda,f) = 0$. By substituting our result for

S_{Id}, we obtain the general expression of the detectivity as a function of various parameters of the MIS device.

Let us now evaluate the spectral density S'(f) of fractional fluctuations in the various dark current noise contributions per cm^2 of transversal detector (or gate) area, including also a numerical example for a MIS infrared detector. For a given detector, this needs to be divided by the area A of the detector to yield the corresponding fractional spectral densities: S(f) =[S'(f)]/A. Fractional fluctuations are dimensionless, so S'(f) will have the dimension of a reciprocal frequency, times a squared length unit which simplifies when we divide by the area of the detector at hand. Let S'_{Id} be the spectral density of fractional fluctuations in the noise caused by quantum 1/f fluctuations in diffusion, S'_{Ir} in bulk recombination, S'_{Is} in surface recombination, and S'_{It} in tunneling. With $m_p^* = 0.55\ m_o$, $m_n^* = 0.02\ m_o$, $\tau_n = 10^{-6}s$, $E_g = 0.1$ eV, $3kT/2 = 0.01$ eV, we obtain for a p-type MIS device with $w_p >> L_d$

$$S'_{Id} = (\alpha_{Hnd} + \alpha_{Hnr})[e/f\tau_n I_d]F(a) = \alpha_{coh}\{e^{1/2}/[f(kT\mu\tau_n)^{1/2}N_p]\}F(a)/a$$

$$=(4.6\ \ 10^{-3}/4fN_p)\ 4\ 10^{-10}C^{1/2}/[10^{-6}s\ 1.5\ 10^5(cm^2/Vs)\ 4\ 10^{-21}\ J]^{1/2}$$

$$=1.8\ \ 10^{-6}cm^2/f,\ [\text{or } 10^{-8}\ cm^2/f, \text{ with } \alpha_p = 2\ 10^{-5} \text{ for incoher. noise}] \quad (200)$$

$$S'_{Ir} = [\alpha_{He}e/f(\tau_{no} + \tau_{po})I_r]\tanh x = [\alpha_{He}e/(feAwn\ \ \tanh x)]\tanh x$$
$$= \alpha_{He}/fAwn_i = 4.6\ 10^{-9}cm^2/f \qquad (x = eV/2kT) \qquad (201)$$

$$S'_{Is} = (4\alpha/3\pi)(2/m^*c^2)[3kT/2 + eU/2 + 0.1Ve]\ [e\ \tanh x\ /f(\tau_{no}+\tau_{po})I_s]$$

$$=(4\alpha/3\pi 0.02)(2/500{,}000)[0.025+0.5+0.5]\ \ [e\ \ \tanh x/feAwn_i(e^x-1)]$$
$$= 7\ 10^{-8}cm^2/f \approx S_{Ib} \qquad (202)$$

$$S'_{Itb} = (4\alpha/3\pi)(E_g + 3kT/2)/m^*c^2 = (4/9.5\ \ 137\ \ 0.02)(0.11/500{,}000)$$
$$= 3.3\ 10^{-8}cm^2/f \qquad (203)$$

$$S'_{Itc} = (4\alpha/3\pi)(E_g + 3kT)/2m^*c^2 = (4/9.5\ \ 137\ \ 0.02)(0.12/10^6)$$
$$= 1.8\ 10^{-8}cm^2/f = S'_{Itsc} \qquad (204)$$

S'_{Id} was calculated in the small bias limit for w_p>>L, but w_p= 0.25 L gives the same result; <u>the incoherent case</u> with a lattice constant of 0.65 nm and Θ = 320 K was also listed above (because a 10 μm thick device is very small, so it may be applicable), and would give 1.8 $10^{-10}cm^2/f$ for a n - type device. Eqs. (201) - (204) would be reduced m_p^* /m_n^* = 27.5 times for n - type devices. We mention that S'_{Is} has been calculated with the inclusion of a term of 10% of the applied gate voltage V into the kinetic energy of the carriers at the surface, and that for the back surface recombination current this term has to be dropped in the similar expression of S'_{Ib}. However, we have neglected this here, because the surface recombination terms will not turn out to be important, as we will see below. The applied gate voltage was taken to be V = 5 V. Using Eq. (4.3) and the current densities evaluated in Eqs. (4.10) - (4.14) to calculate the fraction of each current, we obtain

$$1\ cm^{-2}\ f\ S'_I(f) = (20/132)^2\ 1.8\ 10^{-6} + (10/132)^2\ 4.6\ 10^{-9} + (3.6/132)^2\ 7\ 10^{-8} + (0.01/132)^2\ 3.3\ 10^{-8} + (80/132)^2\ 1.8\ 10^{-8} + (17.5/132)^2\ 1.8\ 10^{-8}$$
$$= 3.67\ 10^{-8} + 2.6\ 10^{-11} + 5.2\ 10^{-11} + 1.9\ 10^{-16} + 6.61\ 10^{-9} + 3.17\ 10^{-10}$$
$$= 4.37\ 10^{-8}, \text{ or for incoherent 1/f noise, } 7.1\ 10^{-9}(p) \text{ and } 3\ 10^{-10}(n) \qquad (205)$$

This value can be used in order to estimate the detectivity of the device in our example. Substituting into Eq. (199), we obtain with a quantum efficiency η = 0.7 and wavelength of λ =10 μm

$$D^*(\lambda,f) = (\eta q\lambda/hc)[S_{Id}(f)]^{-1/2} = [0.7\ 1.6\ 10^{-19}C\ 10^{-5}m/(6.6\ 10^{-34}Js\ 3\ 10^8 m/s)]\ [f/(4.37\ 10^{-8}cm^2\ 1.74\ 10^{-6}A^2/cm^4)]^{1/2} = 2\ 10^7\ (cm\ Hz^{1/2}/w) \times f^{1/2}, \text{ [or for incoherent 1/f noise, } 5\ 10^7\ (p), \text{ and } 2.5\ 10^8\ (n)]. \qquad (206)$$

In conclusion we note that for the relatively large devices which we have considered, most of the quantum 1/f noise comes from fluctuations in diffusion and in the tunneling rate via impurity centers in the band gap. The effective mass of the carriers is present in the denominator of all quantum 1/f noise contributions except the coherent quantum 1/f fluctuation present in the diffusion current of large devices. In smaller

devices the diffusion current will also be given by the conventional quantum 1/f formula which contains the effective mass of the carriers in the denominator. For Umklapp scattering the mass of the carriers in the denominator is even squared. Consequently we expect lower quantum 1/f noise from n - type devices, in which the minority carriers are holes, particularly if the devices are very small, e.g., below 10μm.

E. DISCUSSION AND RECOMMENDATIONS

The transition from coherent state quantum 1/f noise to conventional quantum 1/f noise is particularly interesting, and should be studied experimentally. This is possible with a sequence of devices of smaller and smaller size, and will show a considerable change in noise at a size of the order of 10 m^2. The theory of the transition is not yet well developed. Therefore, this experiment nas particular importance; we do not know if the parameter s is sufficient to characterize the transition, and if the parameter s should not be replaced by a power of s, or by any other function of s. The interpolation formula used here is just a guess, or a speculation guided by the physical understanding of coherent quantum 1/f noise as a collective-field effect, and of conventional quantum 1/f noise as an effect which is not based on the collective field state of the particles, but arises from the individual field of each carrier.

The most interesting component of the recombination current is the surface recombination current which plays a major role in the case of infrared detectors with pn junctions. In the case of MIS devices this role is not so important, as our calculation shows. Nevertheless, one should try to reduce both the concentration of recombination centers and the value of the surface potential jump U. This can be accomplished with careful surface treatment, and with a good passivation layer. SiO_2 layers have been successfully used by Radford and Jones in ion-implanted and double - layer epitaxial HgCdTe photodiodes[34].

In general the larger life time of the carriers in MIS devices, compared to junction devices is due to the absence of the damage inflicted by ion - implantation, or by the heavy doping required in double - layer epitaxial photodiodes. The quantum 1/f noise is inversely proportional to this life

time. Therefore, MIS devices should have lower 1/f noise. On the other hand, 1/f noise present in the applied gate voltage, in the timing of the readout and the value of the readout potential will be added as a 1/f noise source, if it is present. In the present calculation, however, this noise source has not been included.

Any reduction in the concentration of tunneling centers present in the band gap will have a positive effect on quantum 1/f noise. As we have seen in Sec. III.4B, p - type devices should yield less tunneling via band gap centers. The effective mass present in the denominator of the quantum 1/f noise formula in this case should just be the effective mass of the carriers after the tunneling process, i.e., the effective mass of the outgoing carriers emerging from the process we have considered, or the effective mass of the carriers coming in to the process of tunneling toward the centers in the band gap. Here we have considered the tunneling process as the slower process which actually controls the rate of tunneling via band gap states. The capture of carriers by the band gap states is the second part of this compound process and has been considered fast enough, so that it does not limit the rate of the total process. In general, however, both parts of the process have to be considered as a limitation on the rate, and in this case our noise formulae have to be revised through the inclusion of an additional term similar to the recombination noise term.

In the case of very small MIS devices, where only conventional quantum 1/f noise should be present, we may find lower noise in the n - type devices, whose bulk minority carriers are holes with much larger effective masses than the electrons. This may happen in spite of the larger tunneling via band gap centers located right under the surface of these devices.

Finally, we would like to emphasize that the present study has attempted to explain the basic concepts of quantum 1/f noise and to illustrate their application to MIS infrared detectors. Although we have tried to pursue the calculation all the way to the evaluation of the detectivity, the data which we used in the calculation may not be applicable in the practical case at hand, and may have to be replaced with pertinent data in any concrete case.

The author would like to acknowledge the help of M. Belasco, M. Kinch, E. Kelso and R. Balcerak in many discussions on MIS devices and their noise problems.

III.5. QUANTUM 1/f NOISE IN SQUIDS

As we have seen in Sec. II.1-3 above, any cross section or process rate defined for electrically charged particles must fluctuate in time with a 1/f spectral density according to quantum electrodynamics, as a consequence of infrared-divergent coupling to low-frequency photons. This fundamental effect leads to quantum 1/f noise observed in many systems with a small number of carriers, and is also present in the cross sections and process rates which determine the resistance and tunneling rate in Josephson junctions, providing a lower limit of the observed 1/f noise.

In a Josephson junction the normal resistance R_n of the barrier is *proportional to a scattering cross section or transition rate experienced* by the electron in quasiparticle tunneling and by the Cooper pairs below the critical current I_c. Therefore

$$R_n^{-2}S_{Rn}(f) = (4\alpha/3\pi)\ [(\Delta\mathbf{v})^2/c^2Nf)$$

$$= (8\alpha/3\pi)\ (v_F^2/c^2Nf) = 4\ 10^{-14}/f\Omega \qquad (207)$$

where we have approximated $(\Delta\mathbf{v})^2$ with $2v_F^2$, v_F being the Fermi velocity, and the number of carriers N simultaneously present in the barrier of volume Ω (in μ^3) by 10^7, for barriers wider than 10^{-7}cm.

Assuming a linear relationship between the critical current I_c and $G_n = R_n^{-1}$, we obtain similar to Rogers and Buhrman[40], substituting however our quantum 1/f source given by Eq. (207) for R_n, the spectral density of voltage fluctuations

$$S_V(f) = (4/f)\ 10^{-12}(T/3K)[R_s g(V)/(R_s + R_J)]^2$$

$$\times[I_cR_n(I^2/I_c^2 -1)^{-1/2} + g(V)V]^2\Omega^{-1}, \tag{208}$$

where $R_J(V)$ is the junction resistance, R_s the shunt resistance, and $g(V)=R_n/R_J$.

The noise caused in a SQUID by the source considered above can be obtained as the sum of the noise contributions from the two junctions.

The above quantum 1/f results of Eqs.(207) and (208) are in good quantitative agreement with the experimental data.

In conclusion, the fundamental quantum 1/f fluctuations of the cross sections and transition rates which determine the normal resistance have been evaluated in this Section for the case of a Josephson junction. Considering the velocity change in the quantum 1/f formula equal to twice the Fermi velocity and the concentration of carriers in the barrier $10^{19}cm^{-3}$, a spectral density of fractional fluctuations in the normal resistance of the barrier of 4 10^{-14}/f was obtained for a Josephson junction with a volume of the barrier of $10^{-12}cm^3$. These fluctuations are inversely proportional to the barrier volume and result in voltage fluctuations both directly and through the dependence of the critical current on the normal resistance, in good agreement with the experimental data.

IV. SUMMARY OF OPTIMAL DESIGN PRINCIPLES

As a general conclusion of the present study, the following general principles of optimal quantum 1/f noise reduction emerge:

1. Avoidance of coherent state quantum 1/f noise by device size reduction below the coherent limit. This size limit is concentration-dependent, as seen from the expression of the coherence parameter $s = 2e^2N'/mc^2 = 5\ 10^{-13}\ cm^{-1} \times N'$ defined in Eq. (80). $N' = nA$ is the number of carriers per unit length of the device in the direction of current flow. A is the cross-sectional area of the current-carrying device, and n is the concentration of carriers. For $s<<1$ we expect conventional quantum 1/f

noise, while for s>>1 the much larger coherent state quantum 1/f noise is to be expected.

2. Avoid control of the device current or voltage by elementary cross sections or process rates tested by a small number of carriers only. Indeed, the number of carriers interrogating the cross section or process rate appears in the denominator of both the conventional and coherent quantum 1/f noise formulae. In particular avoid current concentrations in bottlenecks, and current inhomogeneities. In junction devices higher lifetimes of the carriers lead to an increase in the number of carriers present in the sample which have tested the current-controlling cross sections, and therefore lead to lower quantum 1/f noise.

3. Avoid control of a device exhibiting conventional quantum 1/f noise through elementary processes which involve large accelerations of the current carriers, or large velocity changes. The squared vector velocity change appears as a factor in the conventional quantum 1/f noise formula. For example, Umklapp scattering, inter-valley and lattice scattering are respectively worst, very bad, and bad, compared with ionized impurity scattering, in terms of the fractional mobility fluctuations which they yield. For a given scattering mechanism, choosing current carriers with a large effective mass will in general reduce the conventional quantum 1/f noise, because for the same momentum transfers this results in smaller accelerations. Bulk recombination control of the current through a pn junction will lead to lower quantum 1/f noise than having the current controlled even in part by surface recombination, because the surface recombination centers are in a high localized field region at the interface between bulk and the passivation layer. Therefore, the best passivation is one which reduces the number and the cross section of the surface recombination centers, while also providing the smallest surface potential jump.

Consequent use of these principles, leads to lower 1/f device noise. The quantum 1/f theory allows for CAD optimization of 1/f device noise suppression.

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APPENDIX I

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